

Women in maths: mathematician cards for IB Maths SL and HL

IB Math Revision · IB Maths SL and HL (AA and AI) · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC is needed for task B.

Starter (5 minutes)

1. 23 is prime. Is $2 \times 23 + 1$ prime?
2. Find the area of a sector of radius 6 and angle $\pi/3$, in terms of π .
3. What happens to $8/(x^2 + 4)$ as x gets very large?
4. Find the gradient of $y = 1/(x^2 + 1)$ at $x = 1$.

Main activity (35 minutes)

Task A: Card 1: Sophie Germain primes

A prime p is a Sophie Germain prime when $2p + 1$ is also prime. For example, 11 is one, because 23 is prime.

- a. List the Sophie Germain primes less than 50. How many are there?
- b. Prove that every Sophie Germain prime bigger than 3 leaves remainder 5 when divided by 6.
- c. Start at 2 and keep applying $p \rightarrow 2p + 1$. How many primes do you get in a row? Which number breaks the chain, and why is it not prime?

Task B: Card 2: the witch of Agnesi

The curve $y = a^3/(x^2 + a^2)$ is known as the witch of Agnesi, after Maria Gaetana Agnesi. Take $a = 2$: $y = 8/(x^2 + 4)$.

- Write down the y -intercept and the equation of the horizontal asymptote.
- Find the coordinates of the points of inflection, giving x exactly.
- Use your GDC to find $\int_{-10}^{10} 8/(x^2 + 4) \, dx$ to 3 significant figures, then try larger limits. Suggest the exact area between the whole curve and the x -axis.

Task C: Card 3: Nightingale's rose diagram

Florence Nightingale is known for polar area diagrams ('rose diagrams'). Each month is a sector with the same angle, and the *area* of the sector shows the number. Use 1 mm^2 for each case, and these made-up numbers.

- A diagram shows 12 months as 12 equal sectors. Find the angle of each sector in radians.
- One month had 150 cases. Find the radius of its sector, to 3 significant figures.
- Another month had 4 times as many cases. By what factor is its radius bigger?
- Explain why drawing the *radius* in proportion to the number would mislead. If the radius were doubled, by what factor would the area grow?

Extension

- Start at 89 and keep applying $p \rightarrow 2p + 1$. How many primes do you get in a row, and which number breaks the chain?
- (HL) The region between $y = 8/(x^2 + 4)$ and the x -axis is rotated 2π about the x -axis. Use your GDC with wide limits to suggest the exact volume, given that it is a multiple of π^2 .