

Women in maths: mathematician cards for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · Teacher answers and notes

The starter is non-calculator. A GDC is needed for task B.

Teacher notes

- Print the three tasks as cards and rotate groups between them.
- The cards name each mathematician only through the maths named after her or that she is known for: invite students to research one of them for homework from a reliable source.
- Card 2 is a good GDC task for AI students; AA students can find the points of inflection by hand.

Starter

1. Yes: 47 is prime

- 47 is not divisible by 2, 3 or 5, and $7^2 = 49 > 47$, so 47 is prime.

2. 6π

- $\frac{1}{2} \times 6^2 \times \pi/3 = 6\pi$

3. It tends to 0

- The denominator grows without limit, so the fraction tends to 0.

4. $-1/2$

- $dy/dx = -2x/(x^2 + 1)^2 = -2/4 = -1/2$

Task A: Card 1: Sophie Germain primes

a. 2, 3, 5, 11, 23, 29, 41: seven

- $2 \rightarrow 5, 3 \rightarrow 7, 5 \rightarrow 11, 11 \rightarrow 23, 23 \rightarrow 47, 29 \rightarrow 59, 41 \rightarrow 83$ are all prime.
- $7 \rightarrow 15, 13 \rightarrow 27, 17 \rightarrow 35, 19 \rightarrow 39, 31 \rightarrow 63, 37 \rightarrow 75, 43 \rightarrow 87, 47 \rightarrow 95$ are not.

b. Proof

- A prime $p > 3$ is not divisible by 2 or 3, so $p = 6k + 1$ or $6k + 5$.
- If $p = 6k + 1$ then $2p + 1 = 12k + 3 = 3(4k + 1)$, a multiple of 3 bigger than 3, so not prime.
- So $p = 6k + 5$.

c. 5 primes (2, 5, 11, 23, 47); $95 = 5 \times 19$

- $2 \rightarrow 5 \rightarrow 11 \rightarrow 23 \rightarrow 47 \rightarrow 95$
- $95 = 5 \times 19$ is not prime.

Task B: Card 2: the witch of Agnesi

a. (0, 2); $y = 0$

- $x = 0$ gives $y = 8/4 = 2$.
- As $x \rightarrow \pm\infty, y \rightarrow 0$.

b. $(\pm 2/\sqrt{3}, 3/2)$

- $y' = -16x(x^2 + 4)^{-2}$ and $y'' = 16(3x^2 - 4)/(x^2 + 4)^3$.
- $y'' = 0$ when $x^2 = 4/3$, so $x = \pm 2/\sqrt{3}$, and y' changes sign there.
- $y = 8/(4/3 + 4) = 8/(16/3) = 3/2$

c. **11.0; the area approaches 4π**

- $\int_{-10}^{10} = 8 \arctan 5 = 10.98\dots$, which is 11.0.
- With limits ± 100 it is 12.49..., and with ± 1000 , 12.55...
- The values approach $4\pi = 12.566\dots$ (exactly, $8 \times \frac{1}{2} \times \pi$).

Task C: Card 3: Nightingale's rose diagram

a. **$\pi/6$**

- $2\pi \div 12 = \pi/6$

b. **23.9 mm**

- $\frac{1}{2}r^2 \times \pi/6 = 150$, so $r^2 = 1800/\pi = 572.9\dots$
- $r = 23.93\dots$, which is 23.9 mm.

c. **2**

- Area is proportional to r^2 , so 4 times the area needs $\sqrt{4} = 2$ times the radius.

d. **4: the eye compares areas, so a doubled number would look four times as big**

- $\text{Area} \propto r^2$: doubling the radius multiplies the area by 4.

Extension

1. **6 primes (89, 179, 359, 719, 1439, 2879); $5759 = 13 \times 443$**

- 89, 179, 359, 719, 1439 and 2879 are all prime (check with your GDC).
- $2 \times 2879 + 1 = 5759 = 13 \times 443$.

2. **$4\pi^2$**

- $\pi \int 64/(x^2 + 4)^2 dx$ over wide limits gets close to 39.478...
- $39.478\dots \div \pi^2 = 4.000\dots$, so the volume is $4\pi^2$.

Every question is original and every answer was recomputed by computer before publishing.