

Valentine's Day maths for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 14 February · Teacher answers and notes

The starter is non-calculator. A GDC is needed for the main activity.

Teacher notes

- Run task B for real: shuffle four name cards with four volunteers a few times before working it out.
- In task B (c), many students think 'exactly three right' is possible: ask why it is not.
- Task C is a good GDC-skills task: students need both halves of the curve and must use $|x|$ correctly.

Starter

1. **9π**

- $\pi \times 3^2 = 9\pi$

2. **24**

- $4 \times 3 \times 2 \times 1 = 24$

3. **$1/4$**

- $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

4. **$x = 8$ or -8**

- $|x| = 4^{3/2} = 8$, so $x = \pm 8$.

Task A: A heart from simple shapes

a. **$100 + 25\pi \approx 179 \text{ cm}^2$**

- Square: $10^2 = 100$. Two semicircles of radius 5 make one circle: $\pi \times 5^2 = 25\pi$.
- $100 + 25\pi = 178.5\dots$, which is 179 cm^2 (3 s.f.).

b. **51.4 cm**

- Two straight sides: 20 cm. Two semicircular arcs make one circle of diameter 10: 10π .
- $20 + 10\pi = 51.41\dots$

c. **16.7 cm**

- Area = $s^2 + \pi(s/2)^2 = s^2(1 + \pi/4)$.
- $s^2 = 500 \div 1.7853\dots = 280.0\dots$, so $s = 16.73\dots$

Task B: Secret card swap

a. **24**

- $4! = 24$

b. **9 ways; $3/8$**

- Call the friends A, B, C, D. The orders with nobody in their own place are BADC, BCDA, BDAC, CADB, CDAB, CDBA, DABC, DCAB, DCBA: 9 of them.
- $9/24 = 3/8$

c. **1/3**

- Choose the lucky friend: 4 ways. The other three must all miss: 2 ways (for example BCA or CAB).
- $4 \times 2 = 8$, and $8/24 = 1/3$.

d. **0.368**

- $1/e = 0.36787\dots$
- The exact probability for 30 people agrees with $1/e$ to many decimal places.

Task C: A heart-shaped curve

a. **(0, 1) and (0, -1)**

- $(y - |x|^{2/3})^2 = 1 - x^2$, so $y - |x|^{2/3} = \pm\sqrt{1 - x^2}$.
- At $x = 0$: $y = \pm 1$.

b. **π**

- Top minus bottom: $(|x|^{2/3} + \sqrt{1 - x^2}) - (|x|^{2/3} - \sqrt{1 - x^2}) = 2\sqrt{1 - x^2}$.
- That is the same as for the circle $x^2 + y^2 = 1$, so the area is the same: $\int_{-1}^1 2\sqrt{1 - x^2} \, dx = \pi$.

c. **(± 0.617 , 1.51)**

- Maximise $y = x^{2/3} + \sqrt{1 - x^2}$ for $0 < x < 1$: the maximum is at $x = 0.6166\dots$, $y = 1.5117\dots$
- By symmetry the other top is at $x = -0.617$.

Extension

1. **1/20; 6 people**

- Two people: the second matches the first with probability $1/20$.
- With k people, $P(\text{no match}) = (20 \times 19 \times \dots \times (21 - k)) \div 20^k$.
- $k = 5$: $P(\text{no match}) = 0.581\dots$, so $P(\text{match}) = 0.419$. $k = 6$: $P(\text{no match}) = 0.436\dots$, so $P(\text{match}) = 0.564 > 1/2$.

Every question is original and every answer was recomputed by computer before publishing.