

Spring egg geometry for IB Maths SL and HL

IB Math Revision · IB Maths SL and HL (AA and AI) · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC is needed for the main activity.

Starter (5 minutes)

1. Find the volume of a sphere of radius 3, in terms of π .
2. Solve $x^2/25 = 1$.
3. Work out $\int_0^2 x^2 dx$.
4. Two similar solids have lengths in the ratio 1 : 2. What is the ratio of their volumes?

Main activity (35 minutes)

Task A: An egg as an ellipse

The outline of an egg is modelled by $x^2/9 + y^2/4 = 1$, in centimetres.

- a. Write down the length and the width of the egg.
- b. Show that the top half of the outline is $y = 2\sqrt{1 - x^2/9}$. Find y when $x = 1.5$, exactly and to 3 significant figures.
- c. Use your GDC to find the area inside the ellipse (twice the area under the top half). Show that it equals 6π , to 3 significant figures.

Task B: The volume of an egg

The top half of the ellipse, $y = 2\sqrt{(1 - x^2/9)}$, is rotated 2π about the x -axis to make a solid egg. (HL by hand; SL with a GDC.)

- Find the volume of the egg, exactly and to 3 significant figures.
- A sphere has the same length as the egg (radius 3 cm). What fraction of the sphere's volume is the egg?
- A hollow chocolate egg has outside surface given by $a = 3$, $b = 2$ and inside surface $a = 2.8$, $b = 1.8$ (cm), where the volume of such an egg is $\frac{4}{3}\pi ab^2$. Find the volume of chocolate, to 3 significant figures.

Task C: A more egg-like egg

Real eggs are fatter at one end. A better model is $y^2 = (4/9)(9 - x^2)(1 + x/6)$, for $-3 \leq x \leq 3$. Plot $y = \pm\sqrt{(4/9)(9 - x^2)(1 + x/6)}$ on your GDC.

- Where does the egg meet the x -axis?
- Find the widest point: the value of x where y is greatest, and that y , each to 3 significant figures.
- The egg is rotated 2π about the x -axis. Show that its volume is exactly the same as the ellipse egg in task B.

Extension

- Every length of the egg in task C is multiplied by 1.5. Find its new volume, exactly and to 3 significant figures.
- (HL) The part of the ellipse $x^2/9 + y^2/4 = 1$ with $x \geq 0$ is rotated 2π about the y -axis instead. Find the volume.