

Spring egg geometry for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · Teacher answers and notes

The starter is non-calculator. A GDC is needed for the main activity.

Teacher notes

- Bring in real eggs and a ruler: students can measure the length and width and compare their own model with the egg.
- Task C (c) is a lovely surprise: the lopsided egg has exactly the same volume because the extra factor adds an odd function.
- AI SL students can do every part with the GDC's numerical integration; AA HL students should do task B by hand.

Starter

1. **36π**

- $\frac{4}{3}\pi \times 27 = 36\pi$

2. **$x = 5$ or -5**

- $x^2 = 25$

3. **$8/3$**

- $[x^3/3]_0^2 = 8/3$

4. **$1 : 8$**

- Volume scale factor = $2^3 = 8$

Task A: An egg as an ellipse

a. **6 cm long, 4 cm wide**

- $y = 0$ gives $x = \pm 3$; $x = 0$ gives $y = \pm 2$.

b. **$\sqrt{3} \approx 1.73$**

- $y^2 = 4(1 - x^2/9)$, and $y \geq 0$ on the top half.
- $x = 1.5$: $y = 2\sqrt{(1 - 0.25)} = 2\sqrt{0.75} = \sqrt{3} = 1.732\dots$

c. **$18.8 \text{ cm}^2 = 6\pi$**

- $2\int_{-3}^3 2\sqrt{(1 - x^2/9)} \, dx = 18.849\dots$
- $6\pi = 18.849\dots$: in general the area of an ellipse is πab .

Task B: The volume of an egg

a. **$16\pi \approx 50.3 \text{ cm}^3$**

- $V = \pi \int_{-3}^3 4(1 - x^2/9) \, dx = 4\pi[x - x^3/27]_{-3}^3 = 4\pi \times 4 = 16\pi$.
- $16\pi = 50.26\dots$

b. **$4/9$**

- Sphere: 36π . $16\pi/36\pi = 4/9$

c. **12.3 cm³**

- Outside: $\frac{4}{3}\pi \times 3 \times 4 = 16\pi$. Inside: $\frac{4}{3}\pi \times 2.8 \times 3.24 = 12.096\pi$.
- $16\pi - 12.096\pi = 3.904\pi = 12.26\dots$

Task C: A more egg-like egg

a. **$x = 3$ and $x = -3$**

- $y = 0$ when $9 - x^2 = 0$ (or $1 + x/6 = 0$, which gives $x = -6$, outside the domain).

b. **$x = 0.646$, $y = 2.06$**

- Maximise $(9 - x^2)(1 + x/6)$: the derivative is zero when $x^2 + 4x - 3 = 0$, so $x = -2 + \sqrt{7} = 0.6457\dots$
- $y = \sqrt{((4/9)(9 - 0.417)(1.1076))} = 2.0555\dots$

c. **16π**

- $V = \pi \int_{-3}^3 (4/9)(9 - x^2)(1 + x/6) dx = (4\pi/9)[\int (9 - x^2) dx + (1/6)\int x(9 - x^2) dx]$.
- The second integral is of an odd function over -3 to 3 , so it is 0. The first is 36.
- $V = (4\pi/9) \times 36 = 16\pi$

Extension

1. **$54\pi \approx 170 \text{ cm}^3$**

- Volume scale factor $1.5^3 = 3.375$.
- $16\pi \times 3.375 = 54\pi = 169.6\dots$

2. **24π**

- $x^2 = 9(1 - y^2/4)$.
- $V = \pi \int_{-2}^2 9(1 - y^2/4) dy = 9\pi[y - y^3/12]_{-2}^2 = 9\pi \times 8/3 = 24\pi$

Every question is original and every answer was recomputed by computer before publishing.