

Pi Day maths for IB Maths SL and HL

IB Math Revision · IB Maths SL and HL (AA and AI) · 14 March · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC is needed for the main activity.

Starter (5 minutes)

1. A point is chosen at random inside a square of side 2. Find the probability that it lies inside the circle of radius 1 drawn in the square.
2. Using $\pi \approx 22/7$, find the circumference of a circle of diameter 7.
3. Work out $1 - 1/3 + 1/5 - 1/7$ as a single fraction.
4. Work out $\int_0^\pi \sin \theta \, d\theta$.

Main activity (35 minutes)

Task A: Monte Carlo π

Darts land at random points (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$. A dart is a 'hit' if $x^2 + y^2 \leq 1$.

- a. Explain why the probability of a hit is $\pi/4$.
- b. A class throws 400 darts (on a computer) and gets 314 hits. Estimate π .
- c. The number of hits in 1000 darts is $X \sim B(1000, \pi/4)$. Find $E(X)$ to the nearest whole number and the standard deviation of X to 3 significant figures.

Task B: Buffon's needle

A needle of length l is dropped on a floor with parallel lines d apart ($l \leq d$). The probability that it crosses a line is $2l/(\pi d)$.

- a. When $l = d$, find the probability that the needle crosses a line, to 3 significant figures.
- b. A class drops 500 needles with $l = d$ and 318 cross a line. Estimate π to 3 significant figures.
- c. If the needle makes angle θ with the lines, it crosses with probability $(l \sin \theta)/d$, and θ is equally likely to be anywhere from 0 to π . Show that the overall probability is $(1/\pi) \int_0^\pi (l \sin \theta)/d \, d\theta = 2l/(\pi d)$.

Task C: Series for π

Two infinite series for π : Leibniz, $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots$, and Nilakantha, $\pi = 3 + 4/(2 \times 3 \times 4) - 4/(4 \times 5 \times 6) + 4/(6 \times 7 \times 8) - \dots$

- a. Use the first 5 terms of the Leibniz series to estimate π , to 3 significant figures.
- b. Use the first 4 terms of the Nilakantha series to estimate π , to 4 decimal places.
- c. Which series converges faster? How many terms of the Leibniz series are needed before $4 \times$ (partial sum) is within 0.01 of π ? (Use your GDC's sequence or sum function.)

Extension

1. A regular 12-sided polygon is drawn inside a circle of diameter 1. Its perimeter is $12 \sin 15^\circ$. Find it to 4 decimal places and compare with π .
2. (HL) Use the substitution $x = \tan \theta$ to show that $\int_0^1 \frac{4}{(1+x^2)} dx = \pi$.