

Pi Day maths for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 14 March · Teacher answers and notes

The starter is non-calculator. A GDC is needed for the main activity.

Teacher notes

- Pi Day is 14 March because 3/14, written month first, matches $\pi = 3.14\dots$
- Run task A or B live: a spreadsheet with $\text{RAND}()$ or a GDC random-number program, or real cocktail sticks on a floor of floorboards.
- Task C (c) shows why the Leibniz series is beautiful but slow: discuss what ‘converges’ means in practice.

Starter

1. $\pi/4$
 - Circle area $\div$ square area = $\pi \div 4$
2. **22**
 - $22/7 \times 7 = 22$
3. **76/105**
 - $(105 - 35 + 21 - 15)/105 = 76/105$
4. **2**
 - $[-\cos \theta]_0^\pi = 1 + 1 = 2$

Task A: Monte Carlo π

- a. $\pi/4$
 - The hits fill a quarter of a circle of radius 1, area $\pi/4$; the square has area 1.
- b. **3.14**
 - $314/400 \approx \pi/4$, so $\pi \approx 4 \times 314/400 = 3.14$
- c. **785; 13.0**
 - $E(X) = 1000 \times \pi/4 = 785.39\dots$
 - $SD = \sqrt{(1000 \times \pi/4 \times (1 - \pi/4))} = \sqrt{168.5\dots} = 12.98\dots$

Task B: Buffon's needle

- a. **$2/\pi \approx 0.637$**
 - $2/(\pi l) = 2/\pi = 0.6366\dots$
- b. **3.14**
 - $318/500 \approx 2/\pi$, so $\pi \approx 2 \times 500/318 = 3.144\dots$
- c. **$2/(\pi d)$**
 - Average $(l \sin \theta)/d$ over $0 \leq \theta \leq \pi$: $(1/\pi) \times (l/d) \times \int_0^\pi \sin \theta \, d\theta$.
 - $\int_0^\pi \sin \theta \, d\theta = 2$, so the probability is $2/(\pi d)$.

Task C: Series for π

a. **3.34**

- $1 - 1/3 + 1/5 - 1/7 + 1/9 = 263/315$
- $\pi \approx 4 \times 263/315 = 3.339\dots$

b. **3.1452**

- $3 + 1/6 - 1/30 + 1/84 = 3.14523\dots$

c. **Nilakantha; 100 terms**

- The error of the Leibniz estimate after n terms is close to $1/n$.
- After 99 terms the error is 0.0101...; after 100 terms it is 0.00999..., so 100 terms.

Extension

1. **3.1058**

- Each side subtends 30° at the centre, so it is $2 \times \frac{1}{2} \times \sin 15^\circ = \sin 15^\circ$.
- $12 \sin 15^\circ = 3.10582\dots$, a little less than π .

2. **π**

- $dx = \sec^2 \theta \, d\theta$ and $1 + \tan^2 \theta = \sec^2 \theta$, so the integral is $\int 4 \, d\theta$.
- Limits 0 and $\pi/4$: $4 \times \pi/4 = \pi$.

Every question is original and every answer was recomputed by computer before publishing.