

Pi Approximation Day puzzles for IB Maths: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 22 July · Teacher answers and notes

The starter is non-calculator. A GDC is useful in the main activity.

Teacher notes

- Pi Approximation Day is 22 July because $22/7$, written day first, is the famous fraction for π .
- This is a short set for the last days of term or summer school: each puzzle stands alone.
- Task B is a nice first look at continued fractions; HL students can explore why the large term 15 makes $22/7$ so good.

Starter

1. **$3 \frac{1}{7}$**
 - $22 = 3 \times 7 + 1$
2. **$16/113$**
 - $355 - 339 = 16$
3. **44**
 - $22/7 \times 14 = 44$
4. **14**
 - $7 \times 2 = 14$

Task A: How good is $22/7$?

- a. **0.00126**
 - $22/7 - \pi = 3.142857... - 3.141592... = 0.0012644...$
- b. **0.0402%**
 - $0.0012644... \div \pi \times 100 = 0.04024...$
- c. **2.53 m**
 - $2 \times 1000 \times (22/7 - \pi) = 2.528...$

Task B: Continued fractions

- a. **$22/7$**
 - $3 + 1/7 = 22/7$
- b. **$333/106$**
 - $7 + 1/15 = 106/15$, so $1/(106/15) = 15/106$.
 - $3 + 15/106 = 333/106$
- c. **$355/113$; 6 decimal places**
 - $15 + 1 = 16$; $7 + 1/16 = 113/16$; $3 + 16/113 = 355/113$.
 - $355/113 = 3.1415929...; \pi = 3.1415926...: they agree to 6 decimal places.$

Task C: The best fraction

a. **22/7**

- The best for each q is the nearest whole number to $q\pi$, over q : $3/1$, $6/2$, $9/3$, $13/4$, $16/5$, $19/6$, $22/7$, $25/8$, $28/9$, $31/10$.
- $22/7$ is the closest, about 0.0013 away.

b. **0.000000267 (2.67×10^{-7})**

- $355/113 - \pi = 0.000000266764\dots$

c. **4740**

- $0.0012644\dots \div 0.00000026676\dots = 4740.1\dots$

Extension

1. **1/497**

- $22/7 - 223/71 = (1562 - 1561)/497 = 1/497$

Every question is original and every answer was recomputed by computer before publishing.