

National Mathematics Day activities for IB Maths

IB Math Revision · IB Maths SL and HL (AA and AI) · 22 December · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC is optional.

Starter (5 minutes)

1. Work out $1^3 + 12^3$.
2. Find $\sum_{r=1}^4 r^3$.
3. In how many ways can 4 be written as a sum of positive integers if order does not matter?
4. Solve $x + y = 19$, $x - y = 1$. Then find $x^3 + y^3$.

Main activity (35 minutes)

Task A: Two ways to make 1729

1729 is the smallest positive integer that is a sum of two positive cubes in two different ways.

- a. Suppose $a^3 + b^3 = 1729$ and $a + b = 13$. Use $a^3 + b^3 = (a + b)((a + b)^2 - 3ab)$ to find ab , and hence a and b .
- b. Repeat with $a + b = 19$.
- c. Explain why $a + b$ must be a factor of 1729, and list the factors of $1729 = 7 \times 13 \times 19$.

Task B: Sums of cubes

$$\sum_{r=1}^n r^3 = (n(n + 1)/2)^2.$$

- a. Find $\sum_{r=1}^{20} r^3$.
- b. Find $\sum_{r=11}^{20} r^3$.
- c. (HL) Prove the formula by induction.

Task C: Partitions and compositions

A partition writes a number as a sum of positive integers where order does not matter. A composition is the same but order matters (so $2 + 1$ and $1 + 2$ are different).

- How many partitions does 5 have?
- How many partitions of 7 have all parts different? How many have all parts odd?
- Explain why 7 has $2^6 = 64$ compositions.

Task D: The birthday magic square as a linear system

Ramanujan was born on 22-12-1887. In this 4×4 magic square the top row is 22 | 12 | 18 | 87, and every row, column and diagonal has the same total.

22	12	18	87
a	b	9	25
10	24	c	16
19	d	23	11

- Write down four linear equations in a , b , c and d (use row 2, row 3, column 2 and the leading diagonal), and solve them.

Extension

- How many positive divisors does $1728 = 2^6 \times 3^3$ have?
- Show that $4104 = 2^3 + 16^3 = 9^3 + 15^3$. Use the method of task A to find the pair for $a + b = 24$.