National Mathematics Day activities for IB Maths: IGCSE / GCSE

IB Math Revision · CBSE Class 9 and Class 10 · 22 December · Student sheet

Name _____ Class _____ Date _____

No calculator. Every answer is a whole number or an exact fraction.

Starter (5 minutes)

1. Work out 12^3.

2. Work out $10^3 + 9^3$.

3. Write 1729 as a product of prime factors. (Hint: try 7.)

4. In how many ways can 4 be written as a sum of positive whole numbers, if the order does not matter? (4 itself counts.)

Main activity (35 minutes)

Task A: The number 1729

1729 is the smallest positive whole number that can be written as the sum of two positive cubes in two different ways.

a. Show that $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

b. Use the identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ to write $9^3 + 10^3$ as a product of two whole numbers.

c. $4104 = 2^3 \times 3^3 \times 19$. Find the HCF and LCM of 1729 and 4104.

Task B: Partitions

A partition of a number is a way of writing it as a sum of positive whole numbers, where the order does not matter. For example, 3 has three partitions: 3, 2 + 1 and 1 + 1 + 1.

a. List all the partitions of 5. How many are there?

b. How many partitions does 6 have?

c. How many partitions of 6 use only different numbers? How many use only odd numbers? What do you notice?

Task C: A birthday magic square

In a magic square every row, every column and both diagonals add up to the same total. Ramanujan was born on 22-12-1887. This 4 × 4 magic square has the date 22 | 12 | 18 | 87 as its top row. Four numbers are missing.

22	**12**	**18**	**87**
?	**17**	**9**	**25**
10	**24**	**?**	**16**
19	**?**	**23**	**?**

a. Find the magic total.

b. Find the four missing numbers.

c. Check that the four corners, and the four numbers in the middle, also add up to the magic total.

Task D: Sums of cubes

$1^3 = 1$, $1^3 + 2^3 = 9$, $1^3 + 2^3 + 3^3 = 36$, …

a. Find $1^3 + 2^3 + 3^3 + 4^3$. What kind of numbers are 1, 9, 36 and your answer?

b. The pattern is $1^3 + 2^3 + \ldots + n^3 = (n(n + 1)/2)^2$. Use it to find $1^3 + 2^3 + \ldots + 12^3$.

c. For which n is the sum 44 100?

Extension

1. $1729 = 7 \times 13 \times 19$, and 7, 13, 19 is an AP with common difference 6. Find the smallest number that is the product of three primes in an AP with common difference 6.

2. Show that 4104 is also a sum of two cubes in two different ways: $4104 = 2^3 + 16^3 = 9^3 + 15^3$.

3. Make your own magic square from your birthday, as in task C. Start with your date as the top row and find a way to complete it. (There is no single answer.)