National Mathematics Day activities for IB Maths: HL

IB Math Revision · IB HL and A Level Further Maths (no calculator) · 22 December · Student sheet

Name _____ Class _____ Date _____

No calculator needed: every answer is exact.

Starter (5 minutes)

1. Find $1^3 + 2^3 + \ldots + 10^3$.

2. How many positive divisors does $1729 = 7 \times 13 \times 19$ have?

3. What is the remainder when 1729 is divided by 4?

4. Find ${}^{10}C_2$.

Main activity (30 minutes)

Task A: Ramanujan's number

$1729 = 1^3 + 12^3 = 9^3 + 10^3$.

a. Prove by induction that $\Sigma\, r^3 = (n(n+1)/2)^2$. Hence find $11^3 + 12^3 + \ldots + 20^3$.

b. What is the largest n for which $1^3 + 2^3 + \ldots + n^3$ is less than 1729?

c. Use $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ with $a = 1$, $b = 12$. Find $a^2 - ab + b^2$, and check that $13 \times$ it is 1729.

Task B: Series

Ramanujan was famous for his infinite series.

a. Find the sum to infinity of $1/(r(r+1)(r+2))$, starting at $r = 1$.

b. Given that $\Sigma\, 1/r^2 = \pi^2/6$, the sum of $1/r^2$ over the odd r only is $k\pi^2$. Find k.

c. Find the coefficient of x^4 in the Maclaurin series of $1/(1 - x - x^2)$.

Extension

1. In how many ways can 7 be written as a sum of different positive whole numbers, ignoring order?