National Mathematics Day activities for IB Maths: Beyond school

IB Math Revision · Beyond school: for adults and maths fans · 22 December · Student sheet

Name _____ Class _____ Date _____

No calculator needed. Every answer works out exactly.

Starter (5 minutes)

1. What is the smallest number that is a sum of two positive cubes in two different ways?

2. $1729 = 7 \times 13 \times 19$. Find the sum of its prime factors.

3. How many partitions does 6 have (ways to write it as a sum of positive whole numbers, ignoring order)?

4. What are the last two digits of 1729^2?

Main activity (30 minutes)

Task A: Taxicab numbers

Ramanujan noticed that 1729 is the smallest sum of two cubes in two ways.

a. $4104 = 2^3 + 16^3$. Find another pair (a, b) with $a < b$ and $a^3 + b^3 = 4104$.

b. How many ordered pairs of positive whole numbers (a, b) have $a^3 + b^3 = 1729$?

c. Find the remainder when 2^{1728} is divided by 1729. (1729 is a Carmichael number.)

Task B: Partitions

Ramanujan and Hardy found a formula for the number of partitions $p(n)$.

a. Find $p(10)$.

b. How many partitions of 10 use only odd parts?

c. How many partitions of 10 use only different parts?

Extension

1. $p(24) = 1575$. Find its remainder when divided by 5, and compare with Ramanujan's rule that $p(5k + 4)$ is always a multiple of 5.