

National Mathematics Day activities for IB Maths: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 22 December · Teacher answers and notes

The starter is non-calculator. A GDC is optional.

Teacher notes

- Facts in this pack: 22 December is National Mathematics Day in India, and Ramanujan was born on 22 December 1887 (source: [Press Information Bureau, Government of India \(PDF\)](#)).
- Task A is a neat use of the symmetric functions $a + b$ and ab : the same idea as the sum and product of the roots of a quadratic.
- For task C, Euler's result (distinct parts = odd parts) is a lovely thing to state; HL students can look up the bijection as an exploration idea.

Starter

1. **1729**
 - $1 + 1728 = 1729$
2. **100**
 - $1 + 8 + 27 + 64 = 100$
3. **5**
 - $4, 3 + 1, 2 + 2, 2 + 1 + 1, 1 + 1 + 1 + 1$
4. **$x = 10, y = 9; 1729$**
 - Add: $2x = 20$, so $x = 10$ and $y = 9$.
 - $1000 + 729 = 1729$

Task A: Two ways to make 1729

- a. **$ab = 12; 1$ and 12**
 - $1729 = 13(169 - 3ab)$, so $169 - 3ab = 133$ and $ab = 12$.
 - a and b are the roots of $t^2 - 13t + 12 = 0$: 1 and 12.
- b. **$ab = 90; 9$ and 10**
 - $1729 = 19(361 - 3ab)$, so $361 - 3ab = 91$ and $ab = 90$.
 - $t^2 - 19t + 90 = 0$ gives 9 and 10.
- c. **1, 7, 13, 19, 91, 133, 247, 1729**
 - $1729 = (a + b)(a^2 - ab + b^2)$ and both brackets are positive integers.
 - The 8 factors are 1, 7, 13, 19, 91, 133, 247 and 1729.

Task B: Sums of cubes

- a. **44 100**
 - $(20 \times 21/2)^2 = 210^2 = 44\,100$
- b. **41 075**
 - $44\,100 - (10 \times 11/2)^2 = 44\,100 - 3025 = 41\,075$

c. **See the steps**

- $n = 1$: $1 = (1 \times 2/2)^2$. True.
- Assume true for $n = k$. Then $\sum_1^{k+1} r^3 = k^2(k+1)^2/4 + (k+1)^3 = (k+1)^2(k^2 + 4k + 4)/4 = ((k+1)(k+2)/2)^2$.
- So true for $n = k + 1$; true for $n = 1$; so true for all positive integers n .

Task C: Partitions and compositions

a. **7**

- 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1

b. **5 and 5**

- Different parts: 7, 6+1, 5+2, 4+3, 4+2+1.
- Odd parts: 7, 5+1+1, 3+3+1, 3+1+1+1+1, 1+1+1+1+1+1+1.
- Both counts are 5. The two counts are always equal (Euler).

c. **64**

- Write 7 as $1 + 1 + 1 + 1 + 1 + 1 + 1$. There are 6 plus signs.
- Each plus sign is either kept or merged, independently: $2^6 = 64$ compositions.

Task D: The birthday magic square as a linear system

a. **$a = 88$, $b = 17$, $c = 89$, $d = 86$**

- The total is $22 + 12 + 18 + 87 = 139$.
- Row 2: $a + b = 105$. Row 3: $c = 89$. Column 2: $b + d = 103$. Diagonal: $b + c = 106$.
- So $b = 17$, $a = 88$, $d = 86$.

Extension

1. **28**

- A divisor is $2^i 3^j$ with $0 \leq i \leq 6$ and $0 \leq j \leq 3$: $7 \times 4 = 28$.

2. **$ab = 135$; 9 and 15**

- $8 + 4096 = 729 + 3375 = 4104$.
- $4104 = 24(576 - 3ab)$, so $576 - 3ab = 171$ and $ab = 135$.
- $t^2 - 24t + 135 = 0$ gives 9 and 15.

Every question is original and every answer was recomputed by computer before publishing.