Halloween maths activities for IB Maths SL and HL: Beyond school: answers

IB Math Revision · Beyond school: for adults and maths fans · 31 October · Teacher answers and notes

No calculator needed. Every answer works out exactly.

Teacher notes

- A stretch set for anyone past school maths: adults, maths clubs and keen sixth-formers. Competition style, no calculator, every answer exact.
- Task A is the Fibonacci numbers in disguise; the extension is the famous napkin-ring problem.

Starter

1. **3**
   - The last digit of 13^n runs 3, 9, 7, 1, 3, … (period 4).
   - 13 = 4 × 3 + 1, so the last digit is 3.
2. **6 spiders**
   - If all 20 were bats there would be 40 legs: 36 too few.
   - Each spider adds 6 legs: 36 ÷ 6 = 6 spiders (and 14 bats).
3. **301**
   - It is one more than a multiple of LCM(2, 3, 4, 5, 6) = 60: 61, 121, 181, 241, 301, …
   - The first of these that divides by 7 is 301 = 7 × 43.
4. **69**
   - For hundreds digit a, count the pairs of digits adding to 13 − a.
   - a = 1 to 9 gives 7, 8, 9, 10, 9, 8, 7, 6 and 5 pairs.
   - Total 69

Task A: The haunted staircase

a. **5 ways**
   - 1+1+1+1, 2+1+1, 1+2+1, 1+1+2, 2+2

b. **89 ways**
   - The first move is a 1-step (then n − 1 steps are left) or a 2-step (then n − 2 are left).
   - 1, 2, 3, 5, 8, 13, 21, 34, 55, 89: the Fibonacci numbers

c. **41 ways**
   - Let $a(n)$ count the climbs ending in a 1-step (or empty) and $b(n)$ those ending in a 2-step: $a(n) = a(n-1) + b(n-1)$ and $b(n) = a(n-2)$.
   - Totals for n = 1 to 10: 1, 2, 3, 4, 6, 9, 13, 19, 28, 41

Task B: Masks and pumpkins

a. **3/8**
   - Count the arrangements of 4 with no one in their own place: 9 of the 24.
   - 9/24 = 3/8

b. ***k* = e (the probability tends to 1/e)**
   - By inclusion–exclusion the probability is $1 - 1 + 1/2! - 1/3! + \ldots \pm 1/n!$.
   - This tends to e^{-1}, so k = e.

c. **149/5000**
   - Equal: 100 pairs. Differ by exactly 1: 2 × 99 = 198 pairs.
   - 298/10 000 = 149/5000

Extension

1. **2048π/3 cm³**
   - The hole is $2\sqrt{(100 - 36)} = 16$ cm long.
   - The ring left is the solid of revolution between $y = -8$ and 8 of $\pi(100 - y^2 - 36)$: $\pi\int(64 - y^2)\,dy = \pi(1024 - 1024/3)$
   - = 2048π/3. (Surprise: it depends only on the 16 cm length of the hole.)

Every question is original and every answer was checked before publishing.