

Halloween maths activities for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 31 October · Teacher answers and notes

The starter is non-calculator. A GDC is useful for the main activity.

Teacher notes

- Ask the class to compare the two zombie models on one GDC screen: the curves agree early on and then separate. That is the whole point of a logistic model.
- Part (c) of the pumpkin task links the derivative of volume to surface area: a nice discussion of why $dV/dr = S$ for a sphere.
- In the trick-or-treat task, students often divide by 0.6 instead of 0.7 for the conditional probability. A tree diagram on the board fixes it.

Starter

1. **$x = 5$**

- $32 = 2^5$, so $x = 5$.

2. **4**

- $3^4 = 81$, so $\log_3 81 = 4$.

3. **0.15**

- For independent events, $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.5 = 0.15$.

4. **5**

- $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Task A: Zombie outbreak: an exponential model

a. **20**

- $Z(0) = 20e^0 = 20$

b. **115**

- $20e^{1.75} = 20 \times 5.7546... = 115.09...$
- 115 zombies

c. **11.2 hours**

- $20e^{0.35t} = 1000$, so $e^{0.35t} = 50$.
- $0.35t = \ln 50$, so $t = \ln 50 \div 0.35 = 11.177...$
- 11.2 hours

d. **1.98 hours**

- $e^{0.35t} = 2$, so $t = \ln 2 \div 0.35 = 1.980...$
- 1.98 hours (3 s.f.)

Task B: A better model: the logistic limit

a. **20**

- $2000 \div (1 + 99) = 20$

b. It approaches 2000: the whole village

- As $t \rightarrow \infty$, $e^{-0.5t} \rightarrow 0$, so $Z \rightarrow 2000 \div 1 = 2000$.
- The model says the number of zombies levels off at the size of the village.

c. 9.19 hours

- $2000 \div (1 + 99e^{-0.5t}) = 1000$ gives $99e^{-0.5t} = 1$.
- $-0.5t = \ln(1/99)$, so $t = 2 \ln 99 = 9.190\dots$
- 9.19 hours

d. 1200

- $2000 \div (1 + 99e^{-5}) = 2000 \div 1.66706\dots = 1199.7\dots$
- 1200 (3 s.f.)

Task C: Pumpkin geometry

a. 14 100 cm³

- $V = \frac{4}{3}\pi \times 15^3 = 4500\pi = 14\,137.1\dots$
- 14 100 cm³

b. 2830 cm²

- $S = 4\pi \times 15^2 = 900\pi = 2827.4\dots$
- 2830 cm²

c. $900\pi \approx 2830 \text{ cm}^3$ per cm: the same as the surface area

- $V = \frac{4}{3}\pi r^3$, so $dV/dr = 4\pi r^2$.
- At $r = 15$ this is 900π , which is exactly the surface area: a thin extra skin of thickness δr adds about $S \times \delta r$ to the volume.

Task D: Trick or treat

a. 0.7

- $0.6 \times 0.9 + 0.4 \times 0.4 = 0.54 + 0.16 = 0.7$

b. $27/35 \approx 0.771$

- $P(\text{pumpkin} | \text{treat}) = 0.54 \div 0.7 = 27/35 = 0.771\dots$

c. 0.383

- $X \sim B(10, 0.7)$.
- $P(X \geq 8) = P(8) + P(9) + P(10) = 0.2335 + 0.1211 + 0.0282 = 0.3828\dots$
- 0.383

Extension

1. 1.72%

- $E(2) = 20e = 54.366\dots$; $Z(2) = 2000 \div (1 + 99e^{-1}) = 53.445\dots$
- Percentage error = $(54.366 - 53.445) \div 53.445 \times 100 = 1.72\%$

2. (a) $\lambda = 3$ (b) 41.8°

- (a) $\lambda = 3$ gives $(1 + 6, 2 - 3, 0 + 6) = (7, -1, 6)$.
- (b) The floor has normal $(0, 0, 1)$. $\sin \theta = |2| \div (\sqrt{4 + 1 + 4} \times 1) = 2/3$.
- $\theta = \arcsin(2/3) = 41.8^\circ$

Every question is original and every answer was recomputed by computer before publishing.