Fibonacci Day maths for IB Maths SL and HL: IGCSE / GCSE

IB Math Revision · IGCSE and GCSE (Higher and Extended) · 23 November · Student sheet

Name _____ Class _____ Date _____

The starter is non-calculator. A calculator is allowed in the main activity.

Starter (5 minutes)

1. Write down the next three terms: 1, 1, 2, 3, 5, 8, ...

2. Write down the next two terms of the Fibonacci-type sequence 3, 4, 7, 11, ...

3. Expand and simplify $(1 + \sqrt{5})^2$.

4. Write 8 : 5 as a decimal, in the form n : 1.

Main activity (35 minutes)

Task A: Fibonacci-type sequences

Each term is the sum of the two terms before it.

a. The first two terms are 2 and 5. Find the 6th term.

b. The first two terms are a and b. Show that the 5th term is $2a + 3b$. If the 5th term is 41 and $a = 4$, find b.

c. A Fibonacci-type sequence has 3rd term 10 and 6th term 42. Find the first two terms.

Task B: The golden ratio

Divide each Fibonacci number by the one before it.

a. Work out 13 ÷ 8, 21 ÷ 13 and 34 ÷ 21, each to 3 decimal places. What do you notice?

b. The ratios get closer to the golden ratio φ, the positive solution of $x^2 = x + 1$. Use the quadratic formula to find φ to 3 decimal places.

c. A golden rectangle has length φ times its width. A golden rectangle is 20 cm wide. Find its length to 1 decimal place.

Task C: Fibonacci squares and the spiral

Squares with sides 1, 1, 2, 3, 5 and 8 cm fit together to make a rectangle.

a. What are the length and width of the rectangle?

b. Show that the total area of the squares equals the area of the rectangle.

c. A quarter circle is drawn in each square, with radius equal to the side. Find the total length of the spiral, to 3 significant figures.

Extension

1. $\varphi = (1 + \sqrt{5})/2$. Show that $\varphi^2 = \varphi + 1$ by working out both sides.

2. Every third Fibonacci number is even. How many of the first 60 Fibonacci numbers are even? Explain why the pattern works.