Fibonacci Day maths for IB Maths SL and HL: IGCSE / GCSE: answers

IB Math Revision · IGCSE and GCSE (Higher and Extended) · 23 November · Teacher answers and notes

The starter is non-calculator. A calculator is allowed in the main activity.

Teacher notes

- Fibonacci Day is 23 November because 11/23, written month first, reads 1, 1, 2, 3.
- Task A (c) is simultaneous equations in disguise: let students find the general terms first.
- Task C works well on squared paper with compasses: the finished spirals make a good display.

Starter

1. **13, 21, 34**
   - 5 + 8 = 13, 8 + 13 = 21, 13 + 21 = 34
2. **18, 29**
   - 7 + 11 = 18, 11 + 18 = 29
3. **$6 + 2\sqrt{5}$**
   - $1 + 2\sqrt{5} + 5 = 6 + 2\sqrt{5}$
4. **1.6 : 1**
   - 8 ÷ 5 = 1.6

Task A: Fibonacci-type sequences

a. **31**
- 2, 5, 7, 12, 19, 31

b. **$b = 11$**
- a, b, $a + b$, $a + 2b$, $2a + 3b$
- $8 + 3b = 41$, so $3b = 33$ and $b = 11$.

c. **4 and 6**
- The terms are a, b, $a + b$, $a + 2b$, $2a + 3b$, $3a + 5b$.
- $a + b = 10$ and $3a + 5b = 42$.
- Substitute $b = 10 - a$: $3a + 50 - 5a = 42$, so $a = 4$ and $b = 6$.

Task B: The golden ratio

a. **1.625, 1.615, 1.619: they get closer to about 1.618**
- 13 ÷ 8 = 1.625
- 21 ÷ 13 = 1.6153...
- 34 ÷ 21 = 1.6190...
- The answers go up and down, getting closer to 1.618...

b. **1.618**
- $x^2 - x - 1 = 0$, so $x = (1 \pm \sqrt{5})/2$.
- The positive solution is $(1 + \sqrt{5})/2 = 1.618$ (3 d.p.).

c. **32.4 cm**
- 20 × 1.6180... = 32.36...
- 32.4 cm (1 d.p.)

Task C: Fibonacci squares and the spiral

a. **13 cm by 8 cm**
- The last square is 8 by 8 and sits beside a 5 by 8 rectangle made from the others, so the rectangle is 8 by (8 + 5) = 8 by 13.

b. **104 cm^2**
- 1 + 1 + 4 + 9 + 25 + 64 = 104
- 8 × 13 = 104

c. **31.4 cm**
- Each quarter circle is ¼ × $2\pi r = \pi r/2$.
- Total = $\pi/2 \times (1 + 1 + 2 + 3 + 5 + 8) = \pi/2 \times 20 = 10\pi = 31.41...$
- 31.4 cm (3 s.f.)

Extension

1. **Both sides equal $(3 + \sqrt{5})/2$**
   - $\varphi^2 = (1 + \sqrt{5})^2/4 = (6 + 2\sqrt{5})/4 = (3 + \sqrt{5})/2$.
   - $\varphi + 1 = (1 + \sqrt{5})/2 + 2/2 = (3 + \sqrt{5})/2$.
2. **20**
   - odd + odd = even, odd + even = odd, even + odd = odd, so the pattern odd, odd, even repeats.
   - 60 ÷ 3 = 20 even numbers.

Every question is original and every answer was checked before publishing.