Fibonacci Day maths for IB Maths SL and HL: HL

IB Math Revision · IB HL and A Level Further Maths (no calculator) · 23 November · Student sheet

Name _____ Class _____ Date _____

No calculator needed: every answer is exact.

Starter (5 minutes)

1. Find the positive solution of $x^2 = x + 1$, exactly.

2. If φ is that solution, find $\varphi^2 - \varphi$.

3. $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$. Find F_{15}.

4. Find the limit of F_{n+1}/F_n as $n \to \infty$.

Main activity (30 minutes)

Task A: Binet's formula

Let $\varphi = (1 + \sqrt{5})/2$ and $\psi = (1 - \sqrt{5})/2$.

a. Find $\varphi + \psi$ and $\varphi\psi$.

b. Prove by induction that $F_n = (\varphi^n - \psi^n)/\sqrt{5}$. Use the formula to check F_{10}.

c. Explain why F_n is the whole number nearest to $\varphi^n/\sqrt{5}$, and use this idea to find F_{20} from the rule.

Task B: Sums and series

Some Fibonacci identities.

a. Prove that $F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$, and find the sum of the first 20 Fibonacci numbers.

b. Use $\Sigma\, F_n x^n = x/(1 - x - x^2)$ to find the sum to infinity of $F_n/2^n$.

c. Find the sum to infinity of $F_n/10^n$, as a fraction.

Extension

1. Cassini's identity says $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$. Find $F_9F_{11} - F_{10}^2$, and explain why two neighbouring Fibonacci numbers never share a factor.