Fibonacci Day maths for IB Maths SL and HL: HL: answers

IB Math Revision · IB HL and A Level Further Maths (no calculator) · 23 November · Teacher answers and notes

No calculator needed: every answer is exact.

Teacher notes

- For IB HL and A Level Further Maths classes. Every answer is exact and the whole set works without a calculator.
- b3 is a nice surprise: 10/89 = 0.011235..., and the Fibonacci numbers appear in its decimal expansion.

Starter

1. **$(1 + \sqrt{5})/2$**
   - $x = (1 \pm \sqrt{5})/2$, and the positive one is $(1 + \sqrt{5})/2$.
2. **1**
   - $\varphi^2 = \varphi + 1$, so $\varphi^2 - \varphi = 1$
3. **610**
   - 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610
4. **$\varphi = (1 + \sqrt{5})/2$**
   - If the ratio tends to L, then $L = 1 + 1/L$, so $L^2 = L + 1$.

Task A: Binet's formula

a. **$\varphi + \psi = 1$ and $\varphi\psi = -1$**
   - $\varphi + \psi = 2/2 = 1$
   - $\varphi\psi = (1 - 5)/4 = -1$

b. **$F_{10} = 55$**
   - φ and ψ satisfy $x^2 = x + 1$, so $\varphi^{n+2} = \varphi^{n+1} + \varphi^n$ (and the same for ψ): the formula obeys the Fibonacci rule.
   - It gives $F_1 = 1$ and $F_2 = 1$, so it holds for all n.
   - $F_{10} = 55$

c. **$F_{20} = 6765$**
   - $|\psi^n/\sqrt{5}| < 1/2$ for every $n \geq 1$, because $|\psi| < 1$.
   - Continuing the sequence: $F_{20} = 6765$

Task B: Sums and series

a. **17710**
   - Induction: adding F_{k+1} to $F_{k+2} - 1$ gives $F_{k+3} - 1$.
   - $F_{22} - 1 = 17711 - 1 = 17710$

b. **2**
   - $x = 1/2$: $(1/2)/(1 - 1/2 - 1/4) = (1/2)/(1/4) = 2$

c. **10/89**
   - $x = 1/10$: $(1/10)/(1 - 1/10 - 1/100) = (1/10)/(89/100) = 10/89$

Extension

1. **1**
   - $34 \times 89 - 55^2 = 3026 - 3025 = 1$
   - Any common factor of F_n and F_{n+1} would divide $(-1)^n$.

Every question is original and every answer was checked before publishing.