Fibonacci Day maths for IB Maths SL and HL: Beyond school

IB Math Revision · Beyond school: for adults and maths fans · 23 November · Student sheet

Name _____ Class _____ Date _____

No calculator needed. Every answer works out exactly.

Starter (5 minutes)

1. In how many ways can 10 be written as an ordered sum of 1s and 2s?

2. Is F_{100} odd or even? Give the remainder when it is divided by 2.

3. Which Fibonacci number greater than 1 is a perfect square?

4. Write 100 as a sum of Fibonacci numbers, no two of them next to each other in the sequence. How many terms do you need?

Main activity (30 minutes)

Task A: Fibonacci remainders

Look at the Fibonacci numbers modulo m (their remainders on dividing by m).

a. Write the Fibonacci numbers modulo 3. After how many terms does the pattern repeat?

b. The pattern modulo 10 (the last digits) repeats every 60 terms. Use this to find the last digit of F_{2026}.

c. Modulo 5, how many of the first 60 Fibonacci numbers are multiples of 5?

Task B: A gcd surprise

A remarkable fact: $\gcd(F_m, F_n) = F_{\gcd(m, n)}$.

a. Find $\gcd(F_{12}, F_{18})$ directly, using $F_{12} = 144$ and $F_{18} = 2584$.

b. Use the fact to find $\gcd(F_{100}, F_{150})$.

c. If F_n is prime then n is prime (apart from $n = 4$). But the converse fails. Find the smallest prime p for which F_p is not prime.

Extension

1. A 1 by 12 path is tiled with 1 by 1 squares and 1 by 2 dominoes. How many tilings use exactly 4 dominoes?