Fibonacci Day maths for IB Maths SL and HL: Beyond school: answers

IB Math Revision · Beyond school: for adults and maths fans · 23 November · Teacher answers and notes

No calculator needed. Every answer works out exactly.

Teacher notes

- A stretch set for anyone past school maths: adults, maths clubs and keen sixth-formers. Competition style, no calculator, every answer exact.
- The Pisano period modulo 10 (60) can be found by hand in a few minutes; it is worth letting solvers discover it.

Starter

1. **89**
   - The counts for 1, 2, 3, … are 1, 2, 3, 5, 8, …: for 10 it is 89.
2. **1 (it is odd)**
   - The pattern is odd, odd, even, repeating every 3.
   - $100 = 3 \times 33 + 1$, so F_{100} is odd.
3. **144**
   - $144 = 12^2$ (and it is the only one bigger than 1).
4. **3 terms**
   - $100 = 89 + 8 + 3$ (take the largest Fibonacci number each time).

Task A: Fibonacci remainders

a. **8 terms**
   - 1, 1, 2, 0, 2, 2, 1, 0, then 1, 1, … again.

b. **3**
   - $2026 = 60 \times 33 + 46$, so F_{2026} ends like F_{46}.
   - $F_{46} = 1836311903$, which ends in 3.

c. **12**
   - F_n is a multiple of 5 exactly when n is a multiple of 5.
   - $60 \div 5 = 12$

Task B: A gcd surprise

a. **8**
   - $144 = 2^4 \times 3^2$ and $2584 = 2^3 \times 17 \times 19$.
   - gcd = 8 = F_6, and gcd(12, 18) = 6

b. **F_{50} = 12586269025**
   - gcd(100, 150) = 50

c. **p = 19**
   - F_3, F_5, F_7, F_{11}, F_{13} and F_{17} (2, 5, 13, 89, 233, 1597) are prime.
   - $F_{19} = 4181 = 37 \times 113$

Extension

1. **70**
   - 4 dominoes and 4 squares make 8 pieces.
   - Choose which 4 of the 8 are dominoes: ${}^8C_4 = 70$

Every question is original and every answer was checked before publishing.