

e Day maths for IB Maths SL and HL

IB Math Revision · IB Maths SL and HL (AA and AI) · 7 February · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC is needed for the main activity.

Starter (5 minutes)

1. Write down the value of $\ln(e^3)$.
2. Solve $e^x = 1$.
3. Differentiate e^{4x} .
4. \$100 is invested at 10% a year compound interest. Find its value after 2 years.

Main activity (35 minutes)

Task A: Compounding more often

\$1000 is invested for one year at a nominal annual interest rate of 6%, compounded n times a year. Give money to 2 decimal places.

- a. Find the value after one year when interest is compounded annually.
- b. Find the value after one year when interest is compounded monthly.
- c. Find the value after one year when interest is compounded daily (365 times).
- d. As n gets very large the value approaches $1000e^{0.06}$ (continuous compounding). Find it.

Task B: The limit that defines e

Let $f(n) = (1 + 1/n)^n$: the value of \$1 after one year at 100% interest compounded n times.

- Find $f(1)$, $f(2)$ and $f(10)$, giving $f(10)$ to 4 decimal places.
- Use a table on your GDC to find the smallest whole number n for which $f(n)$ is within 0.01 of e .
- Show that $\ln f(n) = n \ln(1 + 1/n)$. Find this when $n = 1000$, to 4 decimal places. What does it suggest?

Task C: Continuous growth

The number of bacteria in a dish t hours after the start is modelled by $N = 500e^{0.2t}$.

- Find N when $t = 5$, to the nearest whole number.
- Find the time taken for the number of bacteria to double, to 3 significant figures.
- Find dN/dt when $t = 5$, to 3 significant figures, and show that $dN/dt = 0.2N$ at every time.

Extension

- The Maclaurin series is $e^x = 1 + x + x^2/2! + x^3/3! + \dots$. Use the first six terms with $x = 1$ to estimate e , as a fraction and to 4 decimal places.
- Use the Maclaurin series for $\ln(1 + x)$ to explain why $n \ln(1 + 1/n) \rightarrow 1$ as $n \rightarrow \infty$.