

e Day maths for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · 7 February · Teacher answers and notes

The starter is non-calculator. A GDC is needed for the main activity.

Teacher notes

- e Day is 7 February because 2/7, written month first, matches $e = 2.7...$
- Task A works well as a class race: give each group a different n and collect the answers on the board in order.
- In task C, AI students can find the doubling time from a graph; AA students should solve it with logs.

Starter

1. **3**
 - $\ln$ and e^x undo each other.
2. **$x = 0$**
 - $e^0 = 1$
3. **$4e^{4x}$**
 - Chain rule: the derivative of e^{kx} is ke^{kx} .
4. **\$121**
 - $100 \times 1.1^2 = 121$

Task A: Compounding more often

- a. **\$1060.00**
 - $1000 \times 1.06 = 1060$
- b. **\$1061.68**
 - $1000 \times (1 + 0.06/12)^{12} = 1000 \times 1.005^{12} = 1061.677...$
- c. **\$1061.83**
 - $1000 \times (1 + 0.06/365)^{365} = 1061.831...$
- d. **\$1061.84**
 - $1000e^{0.06} = 1061.836...$
 - Compounding more often helps less and less: daily is only 1 cent short of continuous.

Task B: The limit that defines e

- a. **2, 2.25, 2.5937**
 - $f(1) = 2^1 = 2$
 - $f(2) = 1.5^2 = 2.25$
 - $f(10) = 1.1^{10} = 2.59374...$
- b. **$n = 135$**
 - $f(134) = 2.70820...$, which is 0.0100... below $e = 2.71828...$
 - $f(135) = 2.70827...$, which is 0.0099... below e . So $n = 135$.

c. **0.9995: $\ln f(n) \rightarrow 1$, so $f(n) \rightarrow e$**

- $\ln(1 + 1/n)^n = n \ln(1 + 1/n)$ by the power law of logs.
- $1000 \ln 1.001 = 0.99950\dots$
- $\ln f(n)$ approaches 1, so $f(n)$ approaches $e^1 = e$.

Task C: Continuous growth

a. **1359**

- $500e^1 = 1359.14\dots$

b. **3.47 hours**

- $e^{0.2t} = 2$, so $t = \ln 2 \div 0.2 = 3.4657\dots$

c. **272 bacteria per hour**

- $dN/dt = 500 \times 0.2e^{0.2t} = 100e^{0.2t} = 0.2N$.
- At $t = 5$: $100e = 271.8\dots$, which is 272 (3 s.f.).

Extension

1. **163/60 = 2.7167**

- $1 + 1 + 1/2 + 1/6 + 1/24 + 1/120 = 163/60$
- $163/60 = 2.71666\dots$, which is 2.7167 (4 d.p.); $e = 2.71828\dots$

2. **The limit is 1**

- $\ln(1 + x) = x - x^2/2 + \dots$, so $n \ln(1 + 1/n) = 1 - 1/(2n) + \dots$
- Every term after the 1 tends to 0, so the limit is 1, and $(1 + 1/n)^n \rightarrow e$.

Every question is original and every answer was recomputed by computer before publishing.