Diwali maths activities for IB Maths SL and HL: HL: answers

IB Math Revision · IB HL and A Level Further Maths (no calculator) · Teacher answers and notes

No calculator needed: every answer is exact.

Teacher notes

- For IB HL and A Level Further Maths classes. Every answer is exact and the whole set works without a calculator.
- Task A links rotation by π/4 to multiplication by cis(π/4); students can build the rangoli on the Argand diagram one petal at a time.

Starter

1. **cis(π/3)**
   - Multiplying adds the arguments: π/4 + π/12 = 3π/12 + π/12 = π/3.
2. **5**
   - √(9 + 16) = 5
3. **1023**
   - $2^{10} - 1 = 1023$
4. **2**
   - $[-\cos x]$ from 0 to π = 1 − (−1) = 2

Task A: Rangoli by rotation

a. **√2**
- $|z|^8 = 16$, so $|z| = 16^{1/8} = 2^{1/2} = \sqrt{2}$

b. **1 + i**
- The tips are $\sqrt{2}\,\mathrm{cis}(k\pi/4)$.
- $\sqrt{2}\,\mathrm{cis}(\pi/4) = \sqrt{2}(\sqrt{2}/2 + i\sqrt{2}/2) = 1 + i$

c. **4√2**
- 8 triangles, each with two sides √2 and angle π/4 between them.
- 8 × ½ × √2 × √2 × sin(π/4) = 8 × √2/2 = 4√2

d. **0**
- The tips are spread evenly round a circle centred at 0, so they balance.
- Or: the sum of the roots of $z^8 - 16 = 0$ is minus the coefficient of z^7, which is 0.

Task B: The diya's bowl

a. **4 cm**
- At $y = 4$, $x^2 = 16$, so $x = 4$

b. **32π cm³**
- $V = \pi\int_0^4 x^2\,dy = \pi\int_0^4 4y\,dy$
- $= \pi[2y^2]$ from 0 to 4 = 32π

c. **2√2 cm**
- Oil to depth h: $2\pi h^2 = 16\pi$, so $h^2 = 8$
- $h = 2\sqrt{2}$

Extension

1. **−16**
   - For $z^8 - 16 = 0$ the product of the roots is $(-1)^8 \times (-16) = -16$.
   - Check: the tips pair up as z and $-z$, ...

Every question is original and every answer was checked before publishing.