Diwali maths activities for IB Maths SL and HL: Beyond school: answers

IB Math Revision · Beyond school: for adults and maths fans · Teacher answers and notes

No calculator needed. Every answer works out exactly.

Teacher notes

- A stretch set for anyone past school maths: adults, maths clubs and keen sixth-formers. Competition style, no calculator, every answer exact.
- Task A is a first taste of Burnside's lemma; ask solvers to check 14 by listing the necklaces with 0, 1, 2 and 3 gold lamps.

Starter

1. **20**
   - Each of the 8 corners joins to 5 others by a diagonal: $8 \times 5 \div 2 = 20$
2. **4**
   - The last digits of 2^n run 2, 4, 8, 6 (period 4).
   - $2026 = 4 \times 506 + 2$, so the last digit is 4.
3. **14**
   - The dots make a 3 by 3 grid of unit squares: 9 small, 4 of size 2 and 1 of size 3.
   - $9 + 4 + 1 = 14$
4. **330**
   - Put 4 dividers in the 11 gaps between 12 lamps in a row.
   - ${}^{11}C_4 = 330$

Task A: A necklace of lamps

a. **64**
   - $2^6 = 64$

b. **15**
   - ${}^{6}C_2 = 15$

c. **14**
   - Burnside: average the colourings fixed by each of the 6 rotations.
   - Rotations by 0, 1, 2, 3, 4, 5 places fix 64, 2, 4, 8, 4, 2 colourings.
   - $(64 + 2 + 4 + 8 + 4 + 2) \div 6 = 84 \div 6 = 14$

Task B: Rangoli tiles

a. **5**
   - A tiling starts with one upright tile (2 by 3 left) or two flat tiles (2 by 2 left): $3 + 2 = 5$

b. **89**
   - The counts go 1, 2, 3, 5, 8, ...: the Fibonacci numbers.
   - 2 by 10: 89

c. **11**
   - A 3 by n floor (with n even) has $t(n) = 4t(n - 2) - t(n - 4)$ tilings, with $t(0) = 1$ and $t(2) = 3$.
   - $t(4) = 4 \times 3 - 1 = 11$

Extension

1. **153**
   - $t(6) = 4 \times 11 - 3 = 41$
   - $t(8) = 4 \times 41 - 11 = 153$

Every question is original and every answer was checked before publishing.