

Diwali maths activities for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · Teacher answers and notes

The starter is non-calculator. A GDC is useful for the main activity.

Teacher notes

- AI students can skip the integration in C (b) and use the spherical cap volume as a given formula instead; AA students should integrate.
- In A (b), check students are in radian mode before they find $\sin(\pi/8)$.
- A nice discussion for the extension: why is the sum of the eight points zero? (Symmetry: they balance about the origin.)

Starter

1. **6π**

- $\frac{1}{2} \times 36 \times \pi/3 = 6\pi$

2. **8**

- $s = r\theta = 10 \times 0.8 = 8$

3. **(5, 2)**

- Reflection in $y = x$ swaps the coordinates.

4. **120°**

- $180^\circ - 360^\circ/6 = 120^\circ$

Task A: An eight-point rangoli

a. **$(2\sqrt{2}, 2\sqrt{2})$**

- Rotate by $2\pi/8 = \pi/4$: $(4 \cos \pi/4, 4 \sin \pi/4) = (2\sqrt{2}, 2\sqrt{2})$.

b. **3.06**

- Chord = $2 \times 4 \times \sin(\pi/8) = 8 \sin(\pi/8) = 3.061\dots$

c. **$32\sqrt{2} \approx 45.3$**

- Eight triangles, each $\frac{1}{2} \times 4 \times 4 \times \sin(\pi/4) = 4\sqrt{2}$.
- Area = $8 \times 4\sqrt{2} = 32\sqrt{2} = 45.25\dots$

d. **5.01**

- $16\pi - 32\sqrt{2} = 50.27 - 45.25 = 5.01$

Task B: Rangoli petals

a. **$12\pi - 9\sqrt{3}$**

- Sector: $\frac{1}{2} \times 36 \times 2\pi/3 = 12\pi$. Triangle: $\frac{1}{2} \times 36 \times \sin(2\pi/3) = 9\sqrt{3}$.
- Segment = $12\pi - 9\sqrt{3}$

b. **44.2**

- $2(12\pi - 9\sqrt{3}) = 24\pi - 18\sqrt{3} = 44.22\dots$

Task C: The diya

a. **134 cm³**

- $\frac{1}{2} \times \frac{4}{3}\pi \times 4^3 = 128\pi/3 = 134.04\dots$

b. **$40\pi/3 \approx 41.9 \text{ cm}^3$**

- The oil fills $-4 \leq y \leq -2$.
- $V = \int_{-4}^{-2} \pi(16 - y^2) dy = \pi[16y - y^3/3]_{-4}^{-2}$
- $= \pi[(-32 + 8/3) - (-64 + 64/3)] = 40\pi/3 = 41.9$

c. **5/16**

- $(40\pi/3) \div (128\pi/3) = 40/128 = 5/16$

Task D: Tessellations

a. **$n = 12$**

- $2(180 - 360/n) + 60 = 360$ gives $360/n = 30$, so $n = 12$.

b. **9 sides; no**

- Exterior angle = 40° , so $n = 360 \div 40 = 9$.
- $360 \div 140$ is not a whole number, so it cannot tessellate on its own.

Extension

1. **Sum 0; product $-65\,536$**

- $(4 \operatorname{cis}(k\pi/4))^8 = 4^8 \operatorname{cis}(2k\pi) = 65\,536$, by De Moivre's theorem.
- $z^8 - 65\,536 = 0$ has no z^7 term, so the sum of the roots is 0.
- Product of the roots = $(-1)^8 \times (-65\,536) \div 1 = -65\,536$.

Every question is original and every answer was recomputed by computer before publishing.