

Ada Lovelace Day maths for IB Maths SL and HL

IB Math Revision · IB Maths SL and HL (AA and AI) · Student sheet

Name _____

Class _____

Date _____

The starter is non-calculator. A GDC helps in task B.

Starter (5 minutes)

1. An arithmetic sequence has first term 3 and common difference 5. Find the 20th term.
2. Find the value of $\sum_{r=1}^{10} (2r + 1)$.
3. Find the 6th term of the geometric sequence 2, 6, 18, ...
4. A loop starts with $x = 1$ and repeats the instruction 'replace x by $2x + 1$ ' four times. What is x at the end?

Main activity (35 minutes)

Task A: Trace the algorithm

Step 1: set $n = 1$ and $S = 0$. Step 2: add n^2 to S . Step 3: if $n = 10$, output S and stop; otherwise increase n by 1 and go back to step 2.

- a. What number does the algorithm output?
- b. Write the output in sigma notation. Find the output when ' $n = 10$ ' in step 3 is changed to ' $n = 20$ '.
- c. With the stopping condition ' $n = 20$ ', how many times is step 2 carried out?

Task B: Loops that make sequences

Program P sets $u = 5$, then repeats: 'print u , then add 4 to u '. Program Q sets $u = 1000$, then repeats: 'print u , then multiply u by 0.9'.

- Find the 15th number printed by program P.
- Find the sum of the first 15 numbers printed by program P.
- Which number printed by program Q is the first one below 100? Give its position and its value to 3 significant figures.
- If program Q ran for ever, what would the total of all the numbers it prints approach?

Task C: A recursive rule

A sequence is defined by $u_1 = 1$ and $u_{n+1} = 2u_n + 1$.

- Find u_2 , u_3 , u_4 and u_5 .
- Suggest a formula for u_n and use it to find u_{10} .
- Find $u_1 + u_2 + \dots + u_{10}$.

Extension

- The Bernoulli numbers $B_0, B_1, B_2, \dots$ can be defined by $B_0 = 1$ and, for $m \geq 1$, $\sum_{k=0}^m {}^m C_k B_k = 0$. Use this rule, one value of m at a time, to find B_1, B_2, B_3 and B_4 .
- (HL) Prove by induction that the sequence in task C has $u_n = 2^n - 1$ for every positive integer n .