

Ada Lovelace Day maths for IB Maths SL and HL: answers

IB Math Revision · IB Maths SL and HL (AA and AI) · Teacher answers and notes

The starter is non-calculator. A GDC helps in task B.

Teacher notes

- Ask students to write task A as a flowchart or in pseudocode first: tracing an algorithm by hand is the skill.
- In task B (c), many students give 22: check whether they count the first number printed as number 1 or number 0.
- Ada Lovelace is widely known for her notes on the Analytical Engine; one of them sets out a method for the Bernoulli numbers in extension 1. Keep the lesson about the maths.

Starter

1. **98**

- $u_{20} = 3 + 19 \times 5 = 98$

2. **120**

- $2(1 + 2 + \dots + 10) + 10 = 2 \times 55 + 10 = 120$

3. **486**

- $u_6 = 2 \times 3^5 = 2 \times 243 = 486$

4. **31**

- $1 \rightarrow 3 \rightarrow 7 \rightarrow 15 \rightarrow 31$

Task A: Trace the algorithm

a. **385**

- It adds $1^2 + 2^2 + \dots + 10^2$.
- Using $\sum r^2 = n(n+1)(2n+1)/6$: $10 \times 11 \times 21 \div 6 = 385$

b. **$\sum_{r=1}^{20} r^2 = 2870$**

- $20 \times 21 \times 41 \div 6 = 2870$

c. **20 times**

- Step 2 runs once for each of $n = 1, 2, \dots, 20$: 20 times.

Task B: Loops that make sequences

a. **61**

- The printed numbers are 5, 9, 13, ...: arithmetic with $d = 4$.
- $u_{15} = 5 + 14 \times 4 = 61$

b. **495**

- $S_{15} = 15/2 \times (5 + 61) = 495$

c. **The 23rd number, 98.5**

- The k th number printed is $1000 \times 0.9^{k-1}$.
- $1000 \times 0.9^{k-1} < 100$ gives $0.9^{k-1} < 0.1$, so $k - 1 > \ln 0.1 \div \ln 0.9 = 21.85\dots$
- So $k = 23$, and $1000 \times 0.9^{22} = 98.47\dots$, which is 98.5.

d. **10 000**

- Geometric series with $a = 1000$ and $r = 0.9$.
- $S_{\infty} = 1000 \div (1 - 0.9) = 10\,000$

Task C: A recursive rule

a. **3, 7, 15, 31**

- $2 \times 1 + 1 = 3, 2 \times 3 + 1 = 7, 2 \times 7 + 1 = 15, 2 \times 15 + 1 = 31$

b. **$u_n = 2^n - 1$, so $u_{10} = 1023$**

- Each term is one less than a power of 2: $1 = 2 - 1, 3 = 4 - 1, 7 = 8 - 1, \dots$
- $u_{10} = 2^{10} - 1 = 1023$

c. **2036**

- $\Sigma(2^n - 1) = (2 + 4 + \dots + 2^{10}) - 10$
- The geometric part is $2(2^{10} - 1) = 2046$, so the total is $2046 - 10 = 2036$.

Extension

1. **$B_1 = -1/2, B_2 = 1/6, B_3 = 0, B_4 = -1/30$**

- $m = 1$: $B_0 + 2B_1 = 0$, so $B_1 = -1/2$.
- $m = 2$: $B_0 + 3B_1 + 3B_2 = 0$, so $1 - 3/2 + 3B_2 = 0$ and $B_2 = 1/6$.
- $m = 3$: $1 + 4(-1/2) + 6(1/6) + 4B_3 = 0$, so $B_3 = 0$.
- $m = 4$: $1 + 5(-1/2) + 10(1/6) + 10(0) + 5B_4 = 0$, so $5B_4 = -1/6$ and $B_4 = -1/30$.

2. **Proof**

- Base case: $u_1 = 1 = 2^1 - 1$.
- Assume $u_k = 2^k - 1$ for some k . Then $u_{k+1} = 2(2^k - 1) + 1 = 2^{k+1} - 1$.
- It is true for $n = 1$, and if true for $n = k$ it is true for $n = k + 1$, so by induction it is true for every positive integer n .

Every question is original and every answer was recomputed by computer before publishing.