

Number and algebra

IB Maths AI SL · Knowledge organiser

KEY DEFINITIONS

Standard form

A number written as $a \times 10^k$ with $1 \leq a < 10$ and k an integer.

Compound interest

Interest is added to the balance, so each period's interest is earned on a bigger amount.

Percentage error

$|\text{approximate} - \text{exact}| \div |\text{exact}| \times 100\%$.

Amortisation

Paying off a loan in regular instalments that cover the interest and part of the amount borrowed.

YOU SHOULD BE ABLE TO...

- Convert numbers to and from the form $a \times 10^k$ where $1 \leq a < 10$, and interpret very large and very small quantities in context.
- Recognise an arithmetic sequence from its common difference and use $u_n = u_1 + (n - 1)d$ to find terms in contexts such as rows of seats.
- Compare simple and compound interest, use the compound interest formula with different compounding periods and check answers with the calculator's finance solver.
- Use the finance solver and a spreadsheet to model annuities and regular-payment investments, comparing the outcomes of different saving plans.
- Solve polynomial equations such as $2x^3 - 5x^2 + x + 3 = 0$ with the calculator and decide which roots are valid in a given context.
- Find bounds and percentage error for measured quantities and calculated results, and choose an appropriate degree of accuracy when reporting answers.

COMMON MISTAKES

- ✗ Finance: compound-interest formula with regular payments, TVM signs and periods
- ✗ Sequences: u_n vs S_n vs n , and rounding n up
- ✗ Giving bounds as $[\text{value} - 0.5 \times \text{unit}, \text{value} + 0.5 \times \text{unit}]$ with the wrong unit of accuracy
- ✗ Confusing percentage error with absolute error

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Standard form $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$

Arithmetic sequence **B** $u_n = u_1 + (n - 1)d$

Arithmetic series **B** $S_n = \frac{n}{2}(2u_1 + (n - 1)d) = \frac{n}{2}(u_1 + u_n)$

Common difference $d = u_{n+1} - u_n$

Geometric sequence **B** $u_n = u_1 r^{n-1}$

Geometric series, $r \neq 1$ **B** $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$

Compound interest (k times a year, n years, $r\%$) **B** $FV = PV \left(1 + \frac{r}{100k}\right)^{kn}$

Depreciation at $r\%$ a year $V = V_0 \left(1 - \frac{r}{100}\right)^n$

Exponents & logs **B** $a^x = b \iff x = \log_a b$

Log facts $\log_{10} 10^x = x$, $\ln e^x = x$, $e^{\ln x} = x$

Percentage error **B** $\varepsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$

Bounds: a value rounded to the nearest u lies in $\left[x - \frac{u}{2}, x + \frac{u}{2}\right)$

WORKED EXAMPLE

\$2000 is invested at 3% a year, compounded monthly. Find the value after 5 years, to the nearest cent.

1. $FV = PV(1 + r/(100k))^{(kn)}$ with $k = 12$, $n = 5$

2. $FV = 2000 \times (1 + 0.03/12)^{60} = 2000 \times 1.0025^{60}$

Answer: \$2323.23