

Geometry and trigonometry

IB Maths AI SL · Knowledge organiser

KEY DEFINITIONS

Bearing

A direction measured clockwise from north, written with three figures, such as 045° .

Angle of elevation

The angle measured up from the horizontal to the line of sight.

Perpendicular bisector

The line at right angles to a segment through its midpoint: every point on it is the same distance from both ends.

Voronoi diagram

Splits a plane into cells, each holding the points nearer to one site than to any other.

YOU SHOULD BE ABLE TO...

- Find the distance between two points and the midpoint of a line segment in three dimensions, using the 3D form of Pythagoras' theorem.
- Find the angle between two lines and between a line and a plane in 3D solids by identifying and solving the right-angled triangles inside them.
- Use the sine rule to find missing sides and angles in non-right-angled triangles, in degrees, drawing and labelling a diagram for each problem.
- Find the equation of the perpendicular bisector of two points and use bisectors to construct the cells of a Voronoi diagram for a few sites.
- Update a Voronoi diagram when a new site is added, and solve the toxic waste dump problem by finding the point furthest from every site.
- Solve multi-stage navigation problems with three-figure bearings, using the sine and cosine rules to find distances and directions.

COMMON MISTAKES

- ✗ Voronoi diagrams and bisectors: midpoint, negative reciprocal, vertices
- ✗ Triangles and 3D: wrong rule, wrong angle, missing units
- ✗ Using the wrong right-angled triangle when isolating a 3D angle
- ✗ Forgetting to check whether the GDC is in degree or radian mode

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

3D distance ($\Delta x = x_1 - x_2, \dots$) **B** $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

3D midpoint **B** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone **B** $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, curved $A = \pi r l$

Sphere **B** $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Slant height of a cone $l = \sqrt{r^2 + h^2}$

Sine rule **B** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule **B** $c^2 = a^2 + b^2 - 2ab \cos C$, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area of a triangle **B** $A = \frac{1}{2}ab \sin C$

Arc; sector (θ in degrees) **B** $l = \frac{\theta}{360} \times 2\pi r$, $A = \frac{\theta}{360} \times \pi r^2$

Elevation / depression measured from the horizontal; bearings clockwise from north, 3 figures

Perpendicular bisector of AB : through the midpoint of AB , gradient $-\frac{1}{m_{AB}}$

Voronoi: cell edges lie on perpendicular bisectors of pairs of sites; the point furthest from every site ("toxic waste dump") is at a vertex or on the boundary

WORKED EXAMPLE

In triangle ABC , $a = 7$ cm, $b = 8$ cm and angle $C = 60^\circ$. Find c .

1. Cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$
2. $c^2 = 49 + 64 - 2 \times 7 \times 8 \times 0.5 = 57$

Answer: $c = \sqrt{57} = 7.55$ cm (3 s.f.)