

Functions

IB Maths AI SL · Knowledge organiser

KEY DEFINITIONS

Linear model

$y = mx + c$: y changes by the same amount m for each step of 1 in x .

Exponential model

$y = k a^x + c$: y is multiplied by the same factor a for each step of 1 in x .

Asymptote

A line the graph gets closer and closer to but does not reach, such as $y = c$ for an exponential model.

Domain of a model

The input values that make sense in the context, such as $t \geq 0$ for time.

YOU SHOULD BE ABLE TO...

- Use function notation, find the domain and range of a function from its equation or graph, and interpret them in a modelling context.
- Model growth and decay with functions such as $f(x) = ka^x + c$, identify the asymptote and interpret the parameters in context.
- Understand logarithms as the inverse of exponentials, evaluate $\log_{10}$ and $\ln$ with technology and solve equations such as $5^x = 40$.
- Use technology to fit and analyse cubic models, finding maxima, minima and intersections in context.
- Use exponential models to find half-life and doubling time and solve for time with technology.
- Answer long-response functions questions that combine choosing, fitting and using models, linking each part's answer to the context.

COMMON MISTAKES

- ✗ Exponential and log models: $e^0 = 1$, the equation vs the inequality, reading the model
- ✗ Interpreting in context: parameters, derivatives and justifications
- ✗ Mixing up the perpendicular gradient rule ($m_1 \cdot m_2 = -1$)
- ✗ Reading the zeros of $a(x - p)(x - q)$ as p and $-q$ (or $-p$ and $-q$) instead of p and q

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Gradient; forms of $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + c$, $ax + by + d = 0$, a line **B**
 $y - y_1 = m(x - x_1)$

Parallel;
perpendicular $m_1 = m_2$; $m_1 m_2 = -1$

Quadratic: axis of symmetry **B** $x = -\frac{b}{2a}$

Linear; quadratic models $f(x) = mx + c$; $f(x) = ax^2 + bx + c$

Exponential models $f(x) = ka^x + c$, $f(x) = ka^{-x} + c$,
 $f(x) = ke^{rx} + c$

Direct / inverse variation $f(x) = ax^n$, $n \in \mathbb{Z}$

Cubic model $f(x) = ax^3 + bx^2 + cx + d$

Sinusoidal model $f(x) = a \sin(bx) + d$: amplitude $|a|$,
(degrees) period $\frac{360^\circ}{b}$

Inverse function: swap x and y ; graph reflects in $y = x$; domain of $f^{-1} = \text{range of } f$

Horizontal asymptote of $ka^x + c$ $y = c$

WORKED EXAMPLE

A population is modelled by $P = 500 \times 1.04^t$, where t is in years. Find P when $t = 10$, and the first whole year in which P is more than 1000.

1. $P(10) = 500 \times 1.04^{10} = 740.1\dots$

2. $500 \times 1.04^t = 1000$ gives $1.04^t = 2$, so $t = \ln 2 / \ln 1.04 = 17.7\dots$

Answer: $P(10) \approx 740$, and P first passes 1000 when $t = 18$