

Calculus

IB Maths AI SL · Knowledge organiser

KEY DEFINITIONS

Derivative

The gradient function: how fast y changes as x changes, at each point.

Tangent

The straight line that touches the curve at a point and has the same gradient there.

Optimisation

Finding a maximum or minimum value, usually by solving $f'(x) = 0$.

Trapezoidal rule

Estimates the area under a curve by splitting it into trapezia of equal width.

YOU SHOULD BE ABLE TO...

- Find the average rate of change of a function over an interval as the gradient of a chord, with units, in contexts such as speed.
- Use the sign of $f'(x)$ to find where a function increases or decreases, and sketch the graph of f' from the graph of f .
- Find indefinite integrals of ax^n by reversing differentiation and use a boundary condition, such as a point on the curve, to find the constant c .
- Find the points where a curve has a given gradient and write the equations of the tangent and normal there.
- Interpret definite integrals as accumulated change, such as total water flow from a rate, and find areas with technology.
- Use definite integrals with technology to find a total from a rate, for example a volume from a flow rate.

COMMON MISTAKES

- ✗ Calculus: not differentiating, $f(a)$ for $f'(a)$, and the derivative as a rate
- ✗ Trapezoidal rule and integrals: heights, strip width and dx
- ✗ Forgetting to multiply by the original exponent when differentiating x^n
- ✗ Treating the derivative of a constant as anything other than 0

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Derivative of x^n

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

B

Constant multiples, sums

$$f(x) = ax^n + bx^m \Rightarrow f'(x) = anx^{n-1} + bmx^{m-1}$$

Gradient of tangent at $x = a$

$$f'(a); \text{ tangent } y - f(a) = f'(a)(x - a)$$

Gradient of normal

$$-\frac{1}{f'(a)}$$

Increasing / decreasing

$$f'(x) > 0; \quad f'(x) < 0$$

Stationary points; optimisation

$$f'(x) = 0; \text{ check the sign of } f' \text{ either side}$$

Integral of x^n , $n \neq -1$ **B**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Area, $y \geq 0$ **B**

$$A = \int_a^b y dx$$

Trapezoidal rule, $h = \frac{b-a}{n}$ **B**

$$\int_a^b y dx \approx \frac{1}{2}h[(y_0 + y_n) + 2(y_1 + \dots + y_{n-1})]$$

Boundary condition: find C by substituting a known point into $\int f'(x) dx$

WORKED EXAMPLE

$f(x) = x^3 - 3x$. Find the gradient of the curve at $x = 2$ and the equation of the tangent there.

1. $f'(x) = 3x^2 - 3$, so $f'(2) = 9$

2. $f(2) = 8 - 6 = 2$, so the tangent passes through $(2, 2)$

3. $y - 2 = 9(x - 2)$

Answer: Gradient 9; tangent $y = 9x - 16$