

Statistics and probability

IB Maths AI HL · Knowledge organiser

KEY DEFINITIONS

Poisson distribution

Counts random events in a fixed interval at a constant mean rate m ; its mean and variance are both m .

Transition matrix

Holds the probabilities of moving from each state to each state in one step of a Markov chain.

Confidence interval

A range of values, worked out from a sample, that is likely to contain the population parameter.

Type I error

Rejecting H_0 when it is in fact true; its probability is the significance level.

YOU SHOULD BE ABLE TO...

- Distinguish populations from samples and discrete from continuous data, compare sampling techniques and identify sources of bias.
- Find conditional probabilities from Venn diagrams, trees and tables, and decide whether two events are independent.
- Decide whether a binomial, Poisson or normal model fits a situation, justify the choice and use it to calculate probabilities in context.
- Find the mean and variance of linear combinations of independent random variables, such as the total weight of several items packed together.
- Carry out hypothesis tests for a population mean using z or t with technology, stating hypotheses and concluding in context.
- Carry out χ^2 goodness-of-fit tests to given binomial, Poisson and normal models with technology and conclude in context.

COMMON MISTAKES

- ✗ Hypothesis tests: hypotheses in words, the wrong test, and conclusions that don't follow
- ✗ Distributions: variances add, the normal is continuous, the binomial coefficient
- ✗ Treating a convenience sample as if it were random
- ✗ Using the class boundary instead of the midpoint when estimating the mean from grouped data

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

IQR; mean **B** $IQR = Q_3 - Q_1, \quad \bar{x} = \frac{\sum f_i x_i}{n}$

Outliers $\backslash \text{ or } > Q_3 + 1.5 IQR$

Probability **B** $P(A) = \frac{n(A)}{n(U)}, \quad P(A) + P(A') = 1$

Combined events **B** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent **B** $P(A \cap B) = P(A)P(B)$

Conditional **B** $P(A | B) = \frac{P(A \cap B)}{P(B)}$

Expected value **B** $E(X) = \sum xP(X = x)$

Variance $\text{Var}(X) = E(X^2) - [E(X)]^2$

Binomial pmf $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$

Binomial mean, variance **B** $np, \quad np(1-p)$

Poisson $X \sim \text{Po}(m)$ $P(X = x) = \frac{m^x e^{-m}}{x!}$

Poisson mean, variance **B** $E(X) = m, \quad \text{Var}(X) = m$

Sum of independent Poissons $\text{Po}(m_1) + \text{Po}(m_2) = \text{Po}(m_1 + m_2)$

Linear combinations **B** $E(a_1 X_1 \pm a_2 X_2) = a_1 E(X_1) \pm a_2 E(X_2)$

... independent X_i **B** $\text{Var}(a_1 X_1 \pm a_2 X_2) = a_1^2 \text{Var}(X_1) + a_2^2 \text{Var}(X_2)$

Unbiased variance estimate **B** $s_{n-1}^2 = \frac{n}{n-1} s_n^2$

More on the full formula sheet.

WORKED EXAMPLE

$X \sim \text{Po}(3)$. Find $P(X = 2)$.

1. $P(X = 2) = e^{-3} \times 3^2 / 2!$

2. $= 4.5e^{-3}$

Answer: $P(X = 2) = 0.224$ (3 s.f.)