

Number and algebra

IB Maths AI HL · Knowledge organiser

KEY DEFINITIONS

Matrix

A rectangular array of numbers; an $m \times n$ matrix has m rows and n columns.

Inverse matrix

A^{-1} satisfies $AA^{-1} = I$; a square matrix has an inverse only if its determinant is not zero.

Eigenvalue

A number λ with $Av = \lambda v$ for some non-zero vector v ; find it from $\det(A - \lambda I) = 0$.

Complex number

$z = a + bi$ with $i^2 = -1$; it can also be written in polar form $re^{i\theta}$.

YOU SHOULD BE ABLE TO...

- Convert to and from standard form $a \times 10^k$ and calculate with it in scientific and financial contexts, reading calculator notation correctly.
- Recognise a geometric sequence from its common ratio and use $u_n = u_1 r^{n-1}$ to find terms and when a value is first exceeded.
- Use the finance app to calculate loan repayments, amortization schedules and annuity values, and compare repayment options.
- Plot complex numbers on an Argand diagram and find their modulus and argument, with the argument in radians.
- Find determinants and inverses of 2×2 matrices by hand and larger ones with technology, and solve systems of equations in the form $AX = B$.
- Find the steady state of a regular Markov chain by solving equations or using eigenvectors, and interpret long-term behaviour in context.

COMMON MISTAKES

- ✗ Finance on the GDC: signs, months vs years, P/Y and C/Y, and the rounding asked for
- ✗ Markov chains: columns, the initial state, the right power of T
- ✗ Writing a number in standard form with a coefficient outside $1 \leq a < 10$
- ✗ Giving an error interval as an inequality the wrong way round

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Standard form $a \times 10^k$, $1 \leq a < 10$

Arithmetic **B** $u_n = u_1 + (n-1)d$,
 $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

Geometric **B** $u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$

Sum to infinity, $|r| < 1$ **B** $S_\infty = \frac{u_1}{1 - r}$

Compound interest **B** $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

Percentage error **B** $\varepsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$

Exponents & logs **B** $a^x = b \iff x = \log_a b$

Log laws **B** $\log_a xy = \log_a x + \log_a y$,
 $\log_a \frac{x}{y} = \log_a x - \log_a y$, $\log_a x^m = m \log_a x$

Change of base **B** $\log_b a = \frac{\log_c a}{\log_c b}$

Complex numbers **B** $z = a + bi = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$

Modulus, argument $|z| = \sqrt{a^2 + b^2}$, $\arg z = \theta$,
 $\tan \theta = \frac{b}{a}$ (check quadrant)

Products $|z_1 z_2| = |z_1| |z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

Sinusoids: $A \cos(\omega t) + B \cos(\omega t + \phi) = \text{real part of } (A + Be^{i\phi})e^{i\omega t}$, so amplitudes and phases add as complex numbers

2x2 determinant; inverse **B** $\det A = ad - bc$, $A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

3x3 determinant $\det A = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$

More on the full formula sheet.

WORKED EXAMPLE

Find the eigenvalues of $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.

$$1. \det(A - \lambda I) = (2 - \lambda)^2 - 1 = 0$$

$$2. 2 - \lambda = \pm 1$$

Answer: $\lambda = 1$ or $\lambda = 3$