

Geometry and trigonometry

IB Maths AI HL · Knowledge organiser

KEY DEFINITIONS

Graph

A set of vertices joined by edges; the degree of a vertex is the number of edge ends meeting it.

Eulerian circuit

A closed walk using every edge exactly once; a connected graph has one when every degree is even.

Minimum spanning tree

The cheapest set of edges that joins every vertex with no cycles (Kruskal's or Prim's algorithm).

Adjacency matrix

A table whose entry in row i , column j is the number of edges joining vertex i to vertex j .

YOU SHOULD BE ABLE TO...

- Find the distance between two points and the midpoint of a line segment in three dimensions, using the 3D form of Pythagoras' theorem.
- Use the cosine rule to find a side from two sides and the included angle, and an angle from three sides.
- Find the equation of the perpendicular bisector of two points and use bisectors to construct the cells of a Voronoi diagram for a few sites.
- Use the language of graph theory, find the degree of each vertex and recognise simple, complete and connected graphs.
- Use the nearest-neighbour algorithm to find an upper bound for the travelling salesman problem, starting from different vertices to improve it.
- Build a table of least distances and use the nearest-neighbour and deleted-vertex algorithms to bound the best tour.

COMMON MISTAKES

- ✗ IB notation: AB as a length, vectors in columns, f^{-1} vs f' , the variables asked for
- ✗ Graph algorithms: Prim vs nearest neighbour, no method shown, bounds mixed up
- ✗ Using the 2D distance formula without the extra z -term in 3D
- ✗ Using the whole circle (not the fraction $\theta/360$) when finding an arc length or sector area

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

3D distance (

$$\Delta x = x_1 - x_2, \dots) \quad \mathbf{B} \quad d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

3D midpoint $\mathbf{B} \quad \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$

Pyramid; cone; $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, $A = \pi r l$; $V = \frac{4}{3}\pi r^3$,
sphere $\mathbf{B} \quad A = 4\pi r^2$

Sine, cosine rules; $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $c^2 = a^2 + b^2 - 2ab \cos C$,
area $\mathbf{B} \quad A = \frac{1}{2}ab \sin C$

Arc; sector, degrees $\mathbf{B} \quad l = \frac{\theta}{360}2\pi r$, $A = \frac{\theta}{360}\pi r^2$

Arc; sector, radians $\mathbf{B} \quad l = r\theta$, $A = \frac{1}{2}r^2\theta$

Identities $\mathbf{B} \quad \cos^2 \theta + \sin^2 \theta = 1$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$

Radians; bearings $\pi \text{ rad} = 180^\circ$; bearings clockwise from north

Perpendicular bisectors & Voronoi: cell edges on perpendicular bisectors of sites; largest empty circle at a vertex or on the boundary

Reflection in $y = (\tan \theta)x$ $\mathbf{B} \quad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Stretch $\times k$:
horizontal, vertical; enlargement $\mathbf{B} \quad \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}; \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

Rotation about O by θ : anticlockwise;
clockwise $\mathbf{B} \quad \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}; \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

Area of image $|\det A| \times \text{area of object}$

Composite transformation: B then A is AB

Vector magnitude $\mathbf{B} \quad |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Scalar product; angle $\mathbf{B} \quad \mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3 = |\mathbf{v}||\mathbf{w}| \cos \theta$

More on the full formula sheet.

WORKED EXAMPLE

A connected graph has vertex degrees 2, 4, 3, 3 and 2. How many edges does it have, and does it have an Eulerian circuit?

- The degrees add to twice the number of edges: $2 + 4 + 3 + 3 + 2 = 14$
- Two vertices have odd degree

Answer: 7 edges. No Eulerian circuit, but there is an Eulerian trail between the two odd vertices