

Functions

IB Maths AI HL · Knowledge organiser

KEY DEFINITIONS

Logistic model

Growth that starts nearly exponential and levels off at a carrying capacity L .

Sinusoidal model

$y = a \sin(b(x - c)) + d$ for repeating patterns: amplitude a , period $2\pi/b$, principal axis $y = d$.

Linearising data

Taking logs to turn a power or exponential relationship into a straight line.

Composite transformation

Several transformations applied one after another; the order can change the result.

YOU SHOULD BE ABLE TO...

- Find gradients and intercepts and write equations of lines in the forms $y = mx + c$, $ax + by + d = 0$ and $y - y_1 = m(x - x_1)$.
- Model situations with cubic functions and power models $y = ax^n$, including direct and inverse variation, finding parameters from data.
- Fit quadratic models with technology and use the vertex and zeros to answer projectile and optimisation questions.
- Model growth and decay with exponential functions, identify the asymptote and interpret the parameters in context.
- Interpret the parameters of a logistic model, including the carrying capacity, and sketch logistic curves for given values.
- Fit a logistic model with technology and interpret L , k and x_0 in context.

COMMON MISTAKES

- ✗ HL algebra: product rule, brackets, logs and exact solutions
- ✗ Sketches from the GDC: domain, endpoints, turning points, angle mode
- ✗ Mixing up the perpendicular gradient rule ($m_1 \cdot m_2 = -1$)
- ✗ Misreading the y-intercept from an equation not in $y = mx + c$ form

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Lines **B** $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + c$, $ax + by + d = 0$,
 $y - y_1 = m(x - x_1)$

Perpendicular $m_1 m_2 = -1$

Quadratic: axis of symmetry **B** $x = -\frac{b}{2a}$

Models $f(x) = ka^x + c$, $ke^{rx} + c$; ax^n ;
 $a \sin(bx) + d$ (period $\frac{360^\circ}{b}$)

Logistic model $f(x) = \frac{L}{1 + Ce^{-kx}}$ ($L, C, k > 0$)

Linearising $y = ax^b \Rightarrow \ln y = \ln a + b \ln x$;
 $y = ka^x \Rightarrow \ln y = \ln k + x \ln a$

Composite;
inverse $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$

Transformations: $f(x) + b$ up; $f(x - a)$ right; $pf(x)$ vertical $\times p$;
 $f(qx)$ horizontal $\times \frac{1}{q}$

WORKED EXAMPLE

Data fit $y = a x^b$. A graph of $\ln y$ against $\ln x$ is the straight line $\ln y = 0.7 + 1.5 \ln x$. Find a and b .

1. $\ln y = \ln a + b \ln x$

2. So $b = 1.5$ and $\ln a = 0.7$

Answer: $a = e^{0.7} = 2.01$ (3 s.f.) and $b = 1.5$