

Calculus

IB Maths AI HL · Knowledge organiser

KEY DEFINITIONS

Differential equation

An equation linking a function with its derivatives, such as $dy/dx = ky$.

Euler's method

Steps along the curve with $y_{n+1} = y_n + h \times f(x_n, y_n)$ to estimate a solution.

Slope field

A grid of short line segments showing the gradient dy/dx at each point.

Volume of revolution

$V = \pi \int y^2 dx$ when a region is turned through 2π about the x-axis.

YOU SHOULD BE ABLE TO...

- Compare average and instantaneous rates of change and interpret the derivative as the gradient function of a curve and as a rate of change.
- Find stationary points by solving $f'(x) = 0$ and classify them as local maxima or minima, checking with technology.
- Write differential equations from statements about rates of change, such as a population growing in proportion to its size.
- Find volumes of revolution when a region is rotated 360° about the x-axis or the y-axis, using technology where needed.
- Solve coupled linear systems using the eigenvalues and eigenvectors of the coefficient matrix and write the general solution.
- Use coupled differential equation models for populations, predator-prey interactions and combat, interpreting equilibrium points and long-term behaviour.

COMMON MISTAKES

- ✗ HL algebra: product rule, brackets, logs and exact solutions
- ✗ Volumes of revolution: about the y-axis, π , squaring, limits
- ✗ Confusing average rate of change (a gradient of a chord) with instantaneous rate of change (the derivative)
- ✗ Using the tangent gradient m instead of $-1/m$ for the normal

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Derivatives **B** $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$,
 $(\cos x)' = -\sin x$, $(\tan x)' = \frac{1}{\cos^2 x}$, $(e^x)' = e^x$,
 $(\ln x)' = \frac{1}{x}$

Chain, product, quotient **B** $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

Tangent; normal $y - f(a) = f'(a)(x - a)$; $m_n = -\frac{1}{f'(a)}$

Max / min; inflexion $f' = 0$, $f'' < 0$ / $f'' > 0$;
 $f'' = 0$ with sign change

Related rates $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

Integrals **B** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x} dx = \ln|x| + C$,
 $\int \sin x dx = -\cos x + C$

More integrals **B** $\int \cos x dx = \sin x + C$, $\int \frac{1}{\cos^2 x} dx = \tan x + C$,
 $\int e^x dx = e^x + C$

Linear inside; by inspection $\int f(ax+b) dx = \frac{1}{a}F(ax+b) + C$;
 $\int g'(x)f(g(x)) dx = f(g(x)) + C$

Areas **B** $\int_a^b |y| dx$, $\int_c^d |x| dy$

Volumes of revolution **B** $V = \int_a^b \pi y^2 dx$, $V = \int_c^d \pi x^2 dy$

Trapezoidal rule, $h = \frac{b-a}{n}$ **B** $\int_a^b y dx \approx \frac{1}{2}h[(y_0 + y_n) + 2(y_1 + \dots + y_{n-1})]$

Kinematics **B** $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$; distance $\int_{t_1}^{t_2} |v| dt$

Separable $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x) dx$

Euler's method **B** $y_{n+1} = y_n + h f(x_n, y_n)$, $x_{n+1} = x_n + h$

Euler, coupled systems **B** $x_{n+1} = x_n + h f_1(x_n, y_n, t_n)$,
 $y_{n+1} = y_n + h f_2(x_n, y_n, t_n)$, $t_{n+1} = t_n + h$

Coupled linear system $\dot{x} = Mx$ **B** $x = Ae^{\lambda_1 t} p_1 + Be^{\lambda_2 t} p_2$

More on the full formula sheet.

WORKED EXAMPLE

$dy/dx = x + y$ with $y = 1$ when $x = 0$. Use Euler's method with step 0.1 to estimate y when $x = 0.2$.

1. $y_1 = 1 + 0.1 \times (0 + 1) = 1.1$

2. $y_2 = 1.1 + 0.1 \times (0.1 + 1.1) = 1.22$

Answer: $y(0.2) \approx 1.22$