

Number and algebra

IB Maths AA SL · Knowledge organiser

KEY DEFINITIONS

Arithmetic sequence

Each term is the previous term plus the same number d , the common difference.

Geometric sequence

Each term is the previous term times the same number r , the common ratio.

Logarithm

$\log_a b$ is the power you raise a to in order to get b , so $\log_a b = x$ means $a^x = b$.

Binomial coefficient

nC_r counts the ways to choose r items from n ; these numbers are the coefficients in the expansion of $(a + b)^n$.

YOU SHOULD BE ABLE TO...

- Recognise an arithmetic sequence from its common difference and use $u_n = u_1 + (n - 1)d$ to find a term, the first term or the number of terms.
- Use $S_n = u_1(r^n - 1)/(r - 1)$ to find the sum of a finite geometric series and solve problems for an unknown ratio or number of terms.
- Solve exponential equations by writing both sides with the same base, such as $4^{x+1} = 8^x$, and check answers by substitution.
- Calculate binomial coefficients nC_r with and without technology and use them to expand $(a + b)^n$ for larger n , writing out the first few terms.
- Use the general term ${}^nC_r a^{n-r} b^r$ to find a specified term or coefficient of $(a + b)^n$ without writing out the whole expansion.
- Write clear LHS-to-RHS proofs of results such as the sum of the first n odd numbers or algebraic identities, using correct notation.

COMMON MISTAKES

- Sequences: term vs sum, counting n , and spotting the sequence
- Proof: testing a few values instead of proving the general case
- Confusing common difference d with common ratio r
- Using the arithmetic sum formula for a geometric series

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Arithmetic sequence **B**

$$u_n = u_1 + (n - 1)d$$

Arithmetic series **B**

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d) = \frac{n}{2}(u_1 + u_n)$$

Geometric sequence **B**

$$u_n = u_1 r^{n-1}$$

Geometric series, $r \neq 1$ **B**

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$$

Sum to infinity, $|r| < 1$ **B**

$$S_\infty = \frac{u_1}{1 - r}$$

Compound interest

(k times a year, n years, $r\%$) **B**

$$FV = PV \left(1 + \frac{r}{100k}\right)^{kn}$$

Exponents & logs **B**

$$a^x = b \iff x = \log_a b, \quad a^x = e^{x \ln a},$$

$$\log_a a^x = x = a^{\log_a x}$$

Laws of logs **B**

$$\log_a xy = \log_a x + \log_a y,$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y, \quad \log_a x^m = m \log_a x$$

Change of base **B**

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Special values

$$\log_a 1 = 0, \quad \log_a a = 1, \quad \ln e = 1, \quad e^{\ln x} = x$$

Binomial theorem, $n \in \mathbb{N}$ **B**

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

Binomial coefficient **B**

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

General term

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

WORKED EXAMPLE

A geometric sequence has first term 5 and common ratio 3. Find the sum of the first 6 terms.

$$1. S_6 = u_1(r^6 - 1)/(r - 1) \text{ with } u_1 = 5, r = 3, n = 6$$

$$2. S_6 = 5(3^6 - 1)/(3 - 1) = 5 \times 728 / 2$$

Answer: $S_6 = 1820$