

Geometry and trigonometry

IB Maths AA SL · Knowledge organiser

KEY DEFINITIONS

Radian

The angle at the centre of a circle when the arc is as long as the radius; π radians = 180° .

Unit circle

A circle of radius 1 centred at the origin: the point at angle θ is $(\cos \theta, \sin \theta)$.

Sine rule

In any triangle, $a/\sin A = b/\sin B = c/\sin C$. Use it with a side and its opposite angle.

Ambiguous case

Given two sides and a non-included angle, the sine rule can give two possible triangles.

YOU SHOULD BE ABLE TO...

- Find the distance between two points and the midpoint of a line segment in three dimensions, using the 3D form of Pythagoras' theorem.
- Use the cosine rule to find a side from two sides and the included angle, and an angle from three sides, choosing between the sine and cosine rules.
- Use $A = \frac{1}{2}r^2\theta$ to find the area of a sector, and combine it with $\frac{1}{2}ab \sin C$ to find the area of a segment.
- Use $\cos^2\theta + \sin^2\theta = 1$ and the double-angle identities for $\sin 2\theta$ and $\cos 2\theta$ to find exact values and simplify expressions.
- Read the amplitude, period, principal axis and horizontal shift from a graph or context and write its equation $y = a \sin(b(x - c)) + d$.
- Find the parameters of a sine or cosine model from data or a description and use it to answer questions in context.

COMMON MISTAKES

- ✗ GDC in the wrong angle mode, and degrees in radian formulas
- ✗ Trigonometric equations: one solution only, and exact values not known
- ✗ Using the wrong right-angled triangle when isolating a 3D angle
- ✗ Forgetting to set the calculator to degrees

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

3D distance **B** $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

3D midpoint **B** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone **B** $V = \frac{1}{3}Ah$; $V = \frac{1}{3}\pi r^2 h$, curved $A = \pi r l$

Sphere **B** $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$

Sine rule **B** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule **B** $c^2 = a^2 + b^2 - 2ab \cos C$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Area of a triangle **B** $A = \frac{1}{2}ab \sin C$

Ambiguous case: given a, b, A with A acute, two triangles if $b \sin A < a < b$

Arc; sector (θ in radians) **B** $l = r\theta$, $A = \frac{1}{2}r^2\theta$

Radians $\pi \text{ rad} = 180^\circ$

Identities **B** $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cos^2 \theta + \sin^2 \theta = 1$

Double angle **B** $\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
 $= 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

Exact values at $\sin$: $0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$; $\cos$: reverse order;
 $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ $\tan$: $0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}, \text{undefined}$

Symmetry $\sin(\pi - \theta) = \sin \theta$, $\cos(\pi - \theta) = -\cos \theta$,
 $\cos(-\theta) = \cos \theta$, $\sin(-\theta) = -\sin \theta$

$y = a \sin(b(x - c)) + d$ amplitude $|a|$, period $\frac{2\pi}{b}$, principal axis $y = d$, $b > 0$

$\sin x = k$: $x = \arcsin k$ or $\pi - \arcsin k$, then add $2\pi n$; keep solutions in the interval

WORKED EXAMPLE

A sector has radius 6 cm and angle 1.2 radians. Find its arc length and area.

1. Arc length $s = r\theta = 6 \times 1.2$

2. Area $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 36 \times 1.2$

Answer: Arc = 7.2 cm and area = 21.6 cm²