

Functions

IB Maths AA SL · Knowledge organiser

KEY DEFINITIONS

Domain and range

The domain is the set of inputs a function accepts; the range is the set of outputs it gives.

Composite function

$(f \circ g)(x) = f(g(x))$: apply g first, then f .

Inverse function

f^{-1} undoes f . Its graph is the reflection of the graph of f in the line $y = x$.

Discriminant

For $ax^2 + bx + c$, $\Delta = b^2 - 4ac$ tells you how many real roots there are: two, one (repeated) or none.

YOU SHOULD BE ABLE TO...

- Find the gradient and midpoint of two points and write the equation of a line in gradient-intercept, general and point-gradient forms, such as $y - 3 = 2(x - 1)$.
- Sketch parabolas from their factorised, vertex and standard forms, marking intercepts, the vertex and the axis of symmetry.
- Sketch $y = f(x) + a$ and $y = f(x + a)$ from a given graph, describing each as a translation by a vector and tracking key points.
- Solve equations involving rational functions algebraically and with graphs, such as where $(2x + 1)/(x - 3)$ meets a line, checking solutions against the domain.
- Apply a sequence of transformations to a graph in the correct order and write the equation of the resulting graph.
- Work through multi-part problems that combine transformations, inverse functions and differentiation, using results from earlier parts to answer later ones.

COMMON MISTAKES

- ✗ Answer in the wrong form: asymptotes not as equations, coordinates vs values, unfinished answers
- ✗ Transformations and inverses: incomplete descriptions and f^{-1} notation
- ✗ Mixing up the perpendicular gradient rule ($m_1 \cdot m_2 = -1$)
- ✗ Sign errors completing the square

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Gradient; forms of $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + c$, $ax + by + d = 0$, a line **B** $y - y_1 = m(x - x_1)$

Parallel; perpendicular lines $m_1 = m_2$; $m_1 m_2 = -1$

Quadratic: axis of symmetry **B** $x = -\frac{b}{2a}$

Quadratic formula; discriminant **B** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $\Delta = b^2 - 4ac$

Roots: $\Delta > 0$ two distinct real, $\Delta = 0$ one repeated, $\Delta < 0$ no real roots

Vertex form, vertex (h, k) ; intercept form $y = a(x - h)^2 + k$; $y = a(x - p)(x - q)$

Composite; inverse $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$

Graph of f^{-1} : reflect $y = f(x)$ in $y = x$; domain of $f^{-1} = \text{range of } f$

Translations: $f(x) + b$ up b ; $f(x - a)$ right a

Stretches: $pf(x)$ vertical $\times p$; $f(qx)$ horizontal $\times \frac{1}{q}$

Reflections: $-f(x)$ in the x -axis; $f(-x)$ in the y -axis

$y = \frac{ax+b}{cx+d}$ asymptotes $x = -\frac{d}{c}$, $y = \frac{a}{c}$

Exponential $\leftrightarrow$ log $y = a^x \iff x = \log_a y$; e^x and $\ln x$ are inverses

WORKED EXAMPLE

$f(x) = 2x + 3$ and $g(x) = x^2$. Find $(g \circ f)(1)$ and $f^{-1}(x)$.

1. $(g \circ f)(1) = g(f(1)) = g(5) = 5^2 = 25$

2. Let $y = 2x + 3$, swap x and y : $x = 2y + 3$, so $y = (x - 3)/2$

Answer: $(g \circ f)(1) = 25$ and $f^{-1}(x) = (x - 3)/2$