

Calculus

IB Maths AA SL · Knowledge organiser

KEY DEFINITIONS

Derivative

$f'(x)$ gives the gradient of the curve $y = f(x)$ at each point: the rate of change of y with x .

Stationary point

A point where $f'(x) = 0$: a local maximum, a local minimum or a point of inflection with zero gradient.

Definite integral

$\int_a^b f(x) dx$ gives the signed area between the curve and the x -axis from $x = a$ to $x = b$.

Chain rule

To differentiate a function of a function:
 $dy/dx = dy/du \times du/dx$.

YOU SHOULD BE ABLE TO...

- Estimate the gradient of a curve at a point using chords that get closer and closer, and describe this as a limit.
- Find indefinite integrals of power functions and polynomials by reversing differentiation, always including the constant of integration.
- Differentiate $\sin x$, $\cos x$ and $\tan x$ and expressions built from them, such as $3 \sin x - 2 \cos x$, working in radians.
- Solve optimisation problems in which the function involves trigonometric, exponential or logarithmic terms, justifying that the value found is a maximum or minimum.
- Integrate expressions such as $2x(x^2 + 1)^4$ and $\cos x \cdot e^{\sin x}$ by inspection or by a simple substitution, recognising a function times its derivative.
- Integrate $a(t)$ and $v(t)$ with initial conditions to find velocity and displacement functions.

COMMON MISTAKES

- X** Kinematics: speed vs velocity, change of direction, distance vs displacement
- X** GDC in the wrong angle mode, and degrees in radian formulas
- X** Forgetting to multiply by the original exponent when differentiating x^n
- X** Treating dy/dx of a constant as anything other than 0

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Power rule **B** $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

Standard derivatives **B** $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$,
 $(\ln x)' = \frac{1}{x}$

Chain rule **B** $y = g(u)$, $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Product rule **B** $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

Quotient rule **B** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Linear inside $\frac{d}{dx}e^{kx} = ke^{kx}$, $\frac{d}{dx}\sin kx = k \cos kx$,
 $\frac{d}{dx}\ln(kx) = \frac{1}{x}$

Tangent; normal gradient at $x = a$ $y - f(a) = f'(a)(x - a)$; $m_{\text{normal}} = -\frac{1}{f'(a)}$

Increasing / decreasing $f'(x) > 0$; $f'(x) < 0$

Stationary points $f'(x) = 0$: $f'' < 0$ max, $f'' > 0$ min

Concave up $f'' > 0$; point of inflexion: $f'' = 0$ and f'' changes sign

Integral of x^n , $n \neq -1$ **B** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Standard integrals **B** $\int \frac{1}{x} dx = \ln|x| + C$, $\int \sin x dx = -\cos x + C$,
 $\int \cos x dx = \sin x + C$, $\int e^x dx = e^x + C$

Linear inside $\int f(ax + b) dx = \frac{1}{a}F(ax + b) + C$

Reverse chain rule $\int f'(g(x))g'(x) dx = f(g(x)) + C$

Definite integral $\int_a^b f'(x) dx = f(b) - f(a)$

Area, curve and x -axis **B** $A = \int_a^b |y| dx$

More on the full formula sheet.

WORKED EXAMPLE

Find the stationary points of $y = x^3 - 6x^2 + 9x + 1$ and say what type each is.

- $dy/dx = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0$, so $x = 1$ or $x = 3$
- $y(1) = 5$ and $y(3) = 1$
- $d^2y/dx^2 = 6x - 12$: at $x = 1$ it is $-6 < 0$, at $x = 3$ it is $6 > 0$

Answer: (1, 5) is a local maximum and (3, 1) is a local minimum