

# Geometry and trigonometry

IB Maths AA HL · Knowledge organiser

## KEY DEFINITIONS

### Scalar product

$a \cdot b = |a||b|\cos \theta = a_1b_1 + a_2b_2 + a_3b_3$ ; it is zero when  $a$  and  $b$  are perpendicular.

### Vector equation of a line

$r = a + \lambda b$ :  $a$  is a point on the line and  $b$  its direction.

### Vector product

$a \times b$  is a vector perpendicular to both  $a$  and  $b$ , with length  $|a||b|\sin \theta$ .

### Reciprocal trig functions

$\sec \theta = 1/\cos \theta$ ,  $\operatorname{cosec} \theta = 1/\sin \theta$  and  $\cot \theta = 1/\tan \theta$ .

## YOU SHOULD BE ABLE TO...

- Find the distance between two points and the midpoint of a line segment in three dimensions, using the 3D form of Pythagoras' theorem.
- Use  $\text{area} = \frac{1}{2}ab \sin C$  to find areas and unknown sides or angles, and apply it with the sine and cosine rules to problems in context.
- Solve equations such as  $2 \sin x = 1$  in a given interval, finding every solution with the unit circle or a graph, in degrees and radians.
- Given one trigonometric ratio and the quadrant, find the exact values of the others without finding the angle.
- Write the equation of a plane in vector, parametric and Cartesian forms, using a normal vector found with the vector product.
- Use the compound-angle formulae, the double-angle formula for  $\tan 2\theta$  and the identities linking  $\tan$  with  $\sec$  and  $\cot$  with  $\operatorname{cosec}$  to simplify and prove results.

## COMMON MISTAKES

- ✗ Trigonometric equations: missing solutions (obtuse, general or divided away)
- ✗ Vectors: skew lines without the non-parallel check, and planes
- ✗ Using the wrong right-angled triangle when isolating a 3D angle
- ✗ Forgetting whether the calculator is in radians or degrees

## KEY FORMULAS

**B** = given in the exam (formula booklet or list); learn the rest.

3D distance (

$$\Delta x = x_1 - x_2, \dots) \quad d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

**B**

3D midpoint **B**  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Pyramid; cone;  $V = \frac{1}{3}Ah$ ;  $V = \frac{1}{3}\pi r^2 h$ ,  $A = \pi rl$ ;  $V = \frac{4}{3}\pi r^3$ ,  
sphere **B**  $A = 4\pi r^2$

Sine, cosine rules;  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ,  $c^2 = a^2 + b^2 - 2ab \cos C$ ,  
area **B**  $A = \frac{1}{2}ab \sin C$

Arc; sector (radians) **B**  $l = r\theta$ ,  $A = \frac{1}{2}r^2\theta$

Identities **B**  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ ,  $\cos^2 \theta + \sin^2 \theta = 1$

Reciprocal ratios **B**  $\sec \theta = \frac{1}{\cos \theta}$ ,  $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ ,  $\cot \theta = \frac{1}{\tan \theta}$

Pythagorean **B**  $1 + \tan^2 \theta = \sec^2 \theta$ ,  $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

Compound angles **B**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ ,  
 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

Compound tan **B**  $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double angle **B**  $\sin 2\theta = 2 \sin \theta \cos \theta$ ,  
 $\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$ ,  
 $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

Exact values  $\sin : 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$ ;  $\cos : \text{reverse}$ ;  
 $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$   $\tan : 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}, \text{undef.}$

Ranges  $\arcsin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ,  $\arccos x \in [0, \pi]$ ,  
 $\arctan x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$a \sin(b(x - c)) + d$ ,  
 $b > 0$  amplitude  $|a|$ , period  $\frac{2\pi}{b}$

Vector magnitude **B**  $|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Scalar product **B**  $v \cdot w = v_1w_1 + v_2w_2 + v_3w_3 = |v||w| \cos \theta$

More on the full formula sheet.

## WORKED EXAMPLE

Find the angle between  $a = (1, 2, 2)$  and  $b = (2, -1, 2)$ .

1.  $a \cdot b = 2 - 2 + 4 = 4$

2.  $|a| = 3$  and  $|b| = 3$

3.  $\cos \theta = 4/9$

**Answer:  $\theta = 63.6^\circ$  (3 s.f.)**