

Functions

IB Maths AA HL · Knowledge organiser

KEY DEFINITIONS

Factor theorem

$(x - a)$ is a factor of a polynomial $p(x)$ exactly when $p(a) = 0$.

Vertical asymptote

A line $x = a$ that the graph approaches but never meets, often where a denominator is zero.

Odd and even functions

Even: $f(-x) = f(x)$, symmetric in the y-axis.

Odd: $f(-x) = -f(x)$, rotational symmetry about the origin.

Sum and product of roots

For a polynomial of degree n , the sum and product of its roots can be read from its coefficients.

YOU SHOULD BE ABLE TO...

- Find the gradient and midpoint of two points and write the equation of a line in gradient-intercept, general and point-gradient forms, such as $y - 3 = 2(x - 1)$.
- Solve quadratics with the formula, use the discriminant to count real roots and find unknown constants, and sketch parabolas showing intercepts and the vertex.
- Sketch $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$ and $y = f(ax)$ from a given graph, describing each transformation and tracking key points.
- Sketch rational functions with a quadratic numerator or denominator, finding vertical, horizontal and oblique asymptotes and the intercepts.
- Use the factor and remainder theorems to find factors and remainders of polynomials and unknown coefficients, such as k when $(x - 2)$ is a factor.
- Solve inequalities such as $|2x - 1| > x + 3$ and rational inequalities both graphically and analytically, giving the solution set clearly.

COMMON MISTAKES

- ✗ Graphs and transformations: describing a rational function, and graphs of $[f(x)]^2$
- ✗ Complex numbers and polynomials: conjugate roots, all the roots, general solutions
- ✗ Mixing up the perpendicular gradient rule ($m_1 \cdot m_2 = -1$)
- ✗ Sign errors completing the square

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

Lines **B** $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + c$, $ax + by + d = 0$,
 $y - y_1 = m(x - x_1)$

Perpendicular $m_1 m_2 = -1$

Quadratics **B** $x = -\frac{b}{2a}$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $\Delta = b^2 - 4ac$

$\Delta > 0$: two real roots; $\Delta = 0$: repeated root; $\Delta < 0$: complex conjugate roots

Sum, product of roots of $\sum_{r=0}^n a_r x^r$ **B** $-\frac{a_{n-1}}{a_n}$; $\frac{(-1)^n a_0}{a_n}$

Factor & remainder theorems $p(a) = 0 \iff (x - a)$ is a factor;
remainder of $p(x) \div (x - a)$ is $p(a)$

Odd; even $f(-x) = -f(x)$; $f(-x) = f(x)$

Composite; inverse $(f \circ g)(x) = f(g(x))$; $f(f^{-1}(x)) = x$

Transformations: $f(x) + b$ up b ; $f(x - a)$ right a ; $pf(x)$ vertical $\times p$; $f(qx)$ horizontal $\times \frac{1}{q}$; $-f(x)$, $f(-x)$ reflect in x -, y -axis

Graphs: $|f(x)|$ reflects negative parts up; $f(|x|)$ mirrors $x \geq 0$; $\frac{1}{f(x)}$ has asymptotes at zeros of f

$y = \frac{ax+b}{cx+d}$ asymptotes $x = -\frac{d}{c}$, $y = \frac{a}{c}$

WORKED EXAMPLE

Show that $(x - 1)$ is a factor of $p(x) = x^3 - 2x^2 - 5x + 6$ and factorise $p(x)$ fully.

1. $p(1) = 1 - 2 - 5 + 6 = 0$, so $(x - 1)$ is a factor

2. Divide: $p(x) = (x - 1)(x^2 - x - 6)$

3. $x^2 - x - 6 = (x + 2)(x - 3)$

Answer: $p(x) = (x - 1)(x + 2)(x - 3)$