

Calculus

IB Maths AA HL · Knowledge organiser

KEY DEFINITIONS

Integration by parts

$\int u \, dv = uv - \int v \, du$: use it for a product such as $x e^x$ or $x \sin x$.

Implicit differentiation

Differentiate both sides with respect to x , multiplying by dy/dx each time you differentiate a term in y .

Maclaurin series

Writes $f(x)$ as a power series using the derivatives of f at $x = 0$.

Separable differential equation

$dy/dx = f(x)g(y)$ can be solved by writing $\int dy/g(y) = \int f(x) \, dx$.

YOU SHOULD BE ABLE TO...

- Estimate the gradient of a curve at a point using chords that get closer and closer, and describe this as a limit.
- Set up a function for a quantity to be maximised or minimised, find its stationary points and justify the optimum value in context.
- Differentiate $\sec x$, $\operatorname{cosec} x$, $\cot x$, a^x and $\log_a x$ and combine them with the chain, product and quotient rules.
- Integrate $\sec^2 x$, $1/(a^2 + x^2)$ and $1/\sqrt{a^2 - x^2}$, giving answers in terms of $\tan x$, $\arctan$ and $\arcsin$, and evaluate related definite integrals.
- Choose between inspection, substitution, integration by parts and partial fractions for a given integral, and carry it through.
- Evaluate indeterminate limits using L'Hôpital's rule more than once, or using Maclaurin series.

COMMON MISTAKES

- ✗ Arithmetic with limits, fractions and simultaneous equations on Paper 1
- ✗ "Hence", "deduce" and "exact": using the wrong method
- ✗ Forgetting to multiply by the original exponent when differentiating x^n
- ✗ Treating dy/dx of a constant as anything other than 0

KEY FORMULAS

B = given in the exam (formula booklet or list); learn the rest.

First principles **B** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Basic derivatives $(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$,
B $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$

Trig derivatives $(\tan x)' = \sec^2 x$, $(\sec x)' = \sec x \tan x$,
B $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$,
 $(\cot x)' = -\operatorname{cosec}^2 x$

More derivatives $(a^x)' = a^x \ln a$, $(\log_a x)' = \frac{1}{x \ln a}$
B

Inverse trig **B** $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$,
 $(\arctan x)' = \frac{1}{1+x^2}$

Chain, product, quotient **B** $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$, $(uv)' = uv' + vu'$, $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

Implicit $\frac{d}{dx}(y^n) = ny^{n-1} \frac{dy}{dx}$, $\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$

Related rates $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

Tangent; normal $y - f(a) = f'(a)(x - a)$; $m_n = -\frac{1}{f'(a)}$

Max / min; inflexion $f' = 0$ and $f'' < 0$ / $f'' > 0$;
 $f'' = 0$ with sign change

L'Hôpital, for $\frac{0}{0}$ or $\frac{\infty}{\infty}$ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

Standard integrals $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$, $\int \frac{1}{x} \, dx = \ln|x| + C$,
B $\int \sin x \, dx = -\cos x + C$,
 $\int \cos x \, dx = \sin x + C$, $\int e^x \, dx = e^x + C$

More integrals **B** $\int a^x \, dx = \frac{a^x}{\ln a} + C$, $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$,
 $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$

Learn $\int \sec^2 x \, dx = \tan x + C$,
 $\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$,
 $\int f(ax+b) \, dx = \frac{1}{a} F(ax+b) + C$

$\sin^2 x$, $\cos^2 x$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

By parts **B** $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

More on the full formula sheet.

WORKED EXAMPLE

Find $\int x e^x \, dx$.

- Let $u = x$ and $dv/dx = e^x$, so $du/dx = 1$ and $v = e^x$
- $\int x e^x \, dx = x e^x - \int e^x \, dx$

Answer: $\int x e^x \, dx = x e^x - e^x + C$