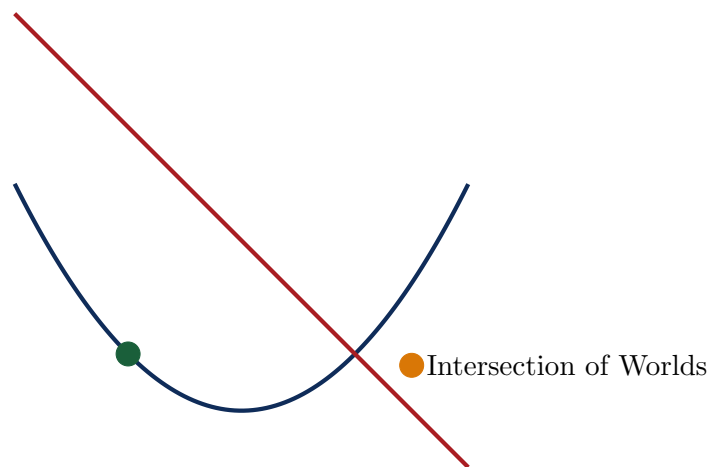


IB MATHEMATICS AI SL

PAPER 2 PRACTICE EXAM

Set 2 | Platinum Tier: Complex Synthesis & Advanced Modelling



Instructions to Candidates

- This practice exam contains five highly rigorous, integrated-response long questions representing the absolute peak difficulty of Paper 2.
- A clean copy of the IB Mathematics Applications and Interpretation formula booklet is required.
- All numerical answers must be given exactly or rounded to 3 significant figures unless specified otherwise (e.g., monetary values to 2 decimal places).
- Section A contains the Exam Questions (each starting on a new page).
- Section B contains the detailed Worked Solutions, Mark Schemes, and Examiner Tips.



SECTION A: EXAM QUESTIONS



Question 1: Compound Interest, Annuities, and Multi-Stage Portfolios [16 Marks]

Lucas is constructing a complex 25-year financial plan divided into two consecutive investment phases to fund a future project.

Phase 1: Accumulation (Years 1 to 10)

For the first 10 years, Lucas makes regular deposits of \$300 at the **end of each month** into an annuity account that pays a nominal annual interest rate of 4.8%, **compounded monthly**.

1. Write down the values of the parameters you would enter into the financial application of your GDC to determine the future value of the annuity at the end of Phase 1:

$N = \underline{\hspace{2cm}}, \quad I\% = \underline{\hspace{2cm}}, \quad PV = \underline{\hspace{2cm}}, \quad PMT = \underline{\hspace{2cm}}, \quad P/Y = \underline{\hspace{2cm}}, \quad C/Y = \underline{\hspace{2cm}}$

[2 marks]

2. Calculate the total accumulated future value in Lucas's annuity at the end of Year 10. Give your answer correct to **two decimal places**.

[2 marks]

Phase 2: Decumulation & Structured Payout (Years 11 to 25)

At the exact start of Year 11, Lucas transfers the entire accumulated balance from Phase 1 into a secure retirement portfolio that earns a nominal annual interest rate of 3.6%, **compounded quarterly**. He intends to make fixed, equal withdrawals of \$ W at the **end of each quarter** for the next 15 years, until the portfolio balance is reduced to exactly zero.

3. State the GDC financial application parameters required to model Phase 2, referencing your answer from part (2). [2 marks]
4. Calculate the exact value of Lucas's quarterly withdrawal, \$ W . [2 marks]
5. Calculate the total interest earned by Lucas across the entire 25-year lifespan of both investment phases combined. [4 marks]

Part C: Inflation-Adjusted Purchasing Power

The average rate of inflation over the 25-year period is a constant 2.4% per year, compounded annually.

6. Find the real purchasing power value (adjusted for inflation) of Lucas's Phase 1 accumulated balance at the end of Year 10. [4 marks]

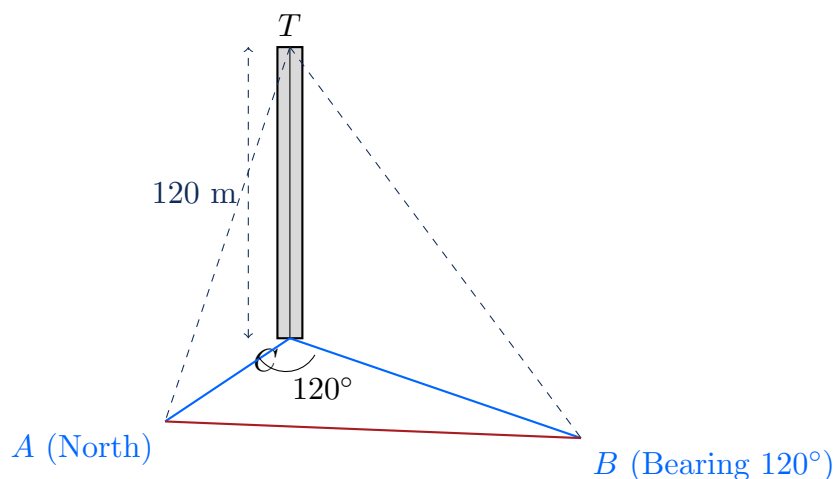


Question 2: Trigonometric Navigation, 3D Bearings, and Cliffside Angles of Depression [16 Marks]

An observer stands at point T at the top of a cliffside lighthouse. The vertical height of point T is exactly 120 m above sea level, represented by the baseline point C directly below the lighthouse.

The observer spots two stationary rescue boats, A and B , in the ocean.

- Boat A is positioned **due North** of C , with an angle of depression from T of 18° .
- Boat B is positioned on a bearing of 120° from C , with an angle of depression from T of 12° .



- Sketch a flat 2D plan-view diagram of the ocean surface, showing points C , A , and B , and write down the exact size of the angle $\hat{ACB}$. [2 marks]
- Calculate the horizontal distance from the base of the cliff C to boat A , giving your answer to the nearest metre. [3 marks]
- Calculate the horizontal distance from the base of the cliff C to boat B , giving your answer to the nearest metre. [3 marks]
- Calculate the straight-line distance between rescue boat A and rescue boat B . [4 marks]
- A rescue helicopter at point H is 500 m directly above boat B . Calculate the angle of elevation of the helicopter H as viewed by a crew member on boat A . [4 marks]



Question 3: Average Cost, Revenue Dynamics, and Quadratic Optimization
[16 Marks]

A hardware firm manufactures custom high-precision titanium bolts. The total daily cost $C(x)$, in euros (EUR), to manufacture x bolts is given by the function:

$$C(x) = 0.2x^2 + 50x + 8000, \quad x \geq 0$$

The market demand dictates that the wholesale selling price $P(x)$, in EUR, per individual bolt drops as production increases, modeled by:

$$P(x) = 250 - 0.5x, \quad 0 \leq x \leq 500$$

1. Find an expression for the average cost per bolt, $A(x)$, and use it to show that the minimum average cost occurs when exactly 200 bolts are produced. **[3 marks]**
2. Determine this minimum average cost per bolt. **[1 mark]**
3. Show that the total daily profit, $\Pi(x)$, in EUR, generated by producing and selling x bolts can be expressed as:

$$\Pi(x) = -0.7x^2 + 200x - 8000$$

[3 marks]

4. Sketch the graph of the profit function $y = \Pi(x)$, showing clearly the coordinates of the turning point (maximum profit) and the x -intercepts. **[3 marks]**
5. State the domain of x for which the firm operates with a **strictly positive profit**, giving your boundaries to the nearest integer. **[3 marks]**
6. Determine the maximum possible daily profit the company can achieve, and the selling price per bolt that Lucas should set to achieve this maximum profit. **[3 marks]**



Question 4: Conditional Probability and Defect Screening [16 Marks]

An advanced micro-sensor assembly line produces units with a baseline historical defect rate of 3%. Every unit is screened through an electronic diagnostic test.

- If a unit is defective (D), the diagnostic test correctly flags it as defective with a probability of 0.98 (true positive).
 - If a unit is not defective (D'), the diagnostic test incorrectly flags it as defective with a probability of x (false positive).
1. Draw a fully labeled tree diagram to represent this screening system, displaying all algebraic path probabilities. [3 marks]
 2. Given that the overall probability that a randomly selected unit is flagged as defective is exactly 0.0779, show that the false positive rate, x , is 0.05. [3 marks]
 3. Calculate the probability that a unit which has been flagged as defective is actually non-defective. [3 marks]
 4. Units that are **not** flagged as defective are approved, packaged, and shipped. Calculate the probability that a shipped unit contains a defect. [3 marks]
 5. A random sample of 15 shipped units is selected. Find the probability that at least one of these units is defective. [4 marks]



Question 5: Hybrid Progressions, Sums, and Crossover Modeling [16 Marks]

Two athletes, Elena and Kenji, begin a 30-week marathon training routine. Their target weekly step goals follow different progression pathways.

Elena's Routine (Arithmetic Progression):

In Week 1, Elena's target is 30 000 steps. Each subsequent week, her target increases by a fixed common difference of 2500 steps.

Kenji's Routine (Geometric Progression):

In Week 1, Kenji's target is 20 000 steps. Each subsequent week, his target increases by a fixed percentage of 12% over the previous week.

1. Calculate Elena's step target in Week 15. [2 marks]
2. Find the total accumulated step target for Elena over the first 15 weeks combined. [3 marks]
3. Calculate Kenji's step target in Week 15, to the nearest integer. [2 marks]
4. Find the total accumulated step target for Kenji over the first 15 weeks combined, to the nearest integer. [3 marks]
5. Using your GDC, determine the first week n in which Kenji's **individual weekly target** strictly exceeds Elena's **individual weekly target**. [3 marks]
6. Using your GDC, determine the first week k in which Kenji's **total accumulated target** strictly exceeds Elena's **total accumulated target**. [3 marks]



SECTION B: WORKED SOLUTIONS



Worked Solution & Mark Distribution - Question 1**1. Accumulation Phase Parameters (GDC):**

Monthly intervals for 10 years ($10 \times 12 = 120$ periods):

$$N = 120, \quad I\% = 4.8, \quad PV = 0, \quad PMT = -300, \\ P/Y = 12, \quad C/Y = 12(\text{M1})\text{A1}$$

Note: Award M1 for an attempt to use a GDC financial solver, and A1 for all entries correct. PMT and FV must have opposite signs.

2. Accumulated Annuity Future Value (FV_1):

Executing the solver on GDC:

$$FV_1 = \$46\,060.52\text{A1A1}$$

Note: Award A1 for the correct value, and A1 for correct rounding to 2 d.p. as required for money.

3. Decumulation Phase Parameters (GDC):

The future value from Phase 1 becomes the initial present value (PV) for Phase 2. Quarterly intervals for 15 years ($15 \times 4 = 60$ periods):

$$N = 60, \quad I\% = 3.6, \quad PV = -46\,060.52, \quad FV = 0, \\ P/Y = 4, \quad C/Y = 4(\text{M1})\text{A1}$$

Note: Award M1 for setting PV as their calculated FV_1 , and A1 for correct period values.

4. Quarterly Withdrawal (W):

Solving for PMT on the GDC:

$$W = \$997.32\text{A1A1}$$

5. Total Interest Earned:

Total deposits made by Lucas (Phase 1) = $300 \times 120 = \$36\,000$. (M1)

Total withdrawals received by Lucas (Phase 2) = $997.32 \times 60 = \$59\,839.20$. (M1)

Total Interest Earned = Total Withdrawals – Total Deposits (M1)

$$\text{Total Interest} = 59\,839.20 - 36\,000 = \$23\,839.20\text{A1}$$

6. Real Value of Year 10 Balance:

Discounting the nominal Phase 1 balance of \$46 060.52 back 10 years using the annual inflation rate of 2.4%:

$$\text{Real Value} = \frac{46\,060.52}{(1 + 0.024)^{10}} (\text{M1})(\text{A1})$$

$$\text{Real Value} = \frac{46\,060.52}{1.26765...} = 36\,335.3402...$$

$$\text{Real Value} = \$36\,335.34\text{A1A1}$$

Examiner Tip & Common Trap

When transitions occur between monthly payments and quarterly payouts, the compounding frequency parameter (C/Y) must change immediately. Underestimating compounding changes



or failing to enter PV and PMT with correct signs relative to cash flow directions remains the most common error on Paper 2.



Worked Solution & Mark Distribution - Question 2

1. **Plan-View Angle $\hat{AC}B$:**

Since boat A is due North (000°) and boat B is on a bearing of 120° :

$$\hat{AC}B = 120^\circ - 0^\circ = 120^\circ \text{ A1A1}$$

Note: Award A1 for the sketch showing points correctly arranged, and A1 for stating 120° .

2. **Horizontal Distance CA :**

In right-angled triangle TCA , the angle of depression is 18° , which equals the angle of elevation $\hat{TAC} = 18^\circ$ (alternate interior angles):

$$\tan(18^\circ) = \frac{120}{CA} \implies CA = \frac{120}{\tan(18^\circ)} \text{ (M1)(A1)}$$

$$CA = 369.324... \implies CA \approx 369 \text{ m A1}$$

3. **Horizontal Distance CB :**

Similarly, in right-angled triangle TCB , the angle of elevation $\hat{TCB} = 12^\circ$:

$$\tan(12^\circ) = \frac{120}{CB} \implies CB = \frac{120}{\tan(12^\circ)} \text{ (M1)(A1)}$$

$$CB = 564.556... \implies CB \approx 565 \text{ m A1}$$

4. **Straight-Line Distance AB :**

Using the non-right-angled triangle ACB on the ocean floor with sides $CA \approx 369.32 \text{ m}$, $CB \approx 564.56 \text{ m}$, and included angle $\hat{ACB} = 120^\circ$:

$$AB^2 = CA^2 + CB^2 - 2(CA)(CB) \cos(120^\circ) \text{ (M1)}$$

$$AB^2 = (369.324)^2 + (564.556)^2 - 2(369.324)(564.556) \cos(120^\circ) \text{ (A1)}$$

$$AB^2 = 136\,397 + 318\,728 - 2(208\,508)(-0.5)$$

$$AB^2 = 455\,125 + 208\,508 = 663\,633 \text{ (A1)}$$

$$AB = \sqrt{663\,633} = 814.636... \implies AB \approx 815 \text{ m A1}$$

5. **Angle of Elevation from A to Helicopter H :**

Helicopter H is vertically above B , forming right-angled triangle ABH where $AB \approx 814.64 \text{ m}$ is the horizontal base, and the height $BH = 500 \text{ m}$ is the vertical height:

$$\tan(\theta) = \frac{\text{Height}}{\text{Base}} = \frac{BH}{AB} = \frac{500}{814.636...} \text{ (M1)(A1)}$$

$$\tan(\theta) = 0.61376... \text{ (A1)}$$

$$\theta = \tan^{-1}(0.61376...) = 31.538^\circ \implies \theta \approx 31.5^\circ \text{ A1}$$

Examiner Tip & Common Trap

3D navigation questions require synthesis across different geometric planes. The vertical triangles (TCA and TCB) are right-angled, but the horizontal floor triangle (ACB) is non-right-angled. Do not apply right-angled SOH CAH TOA or Pythagoras directly to horizontal segments like AB .



Worked Solution & Mark Distribution - Question 3

1. Average Cost Optimization:

$$A(x) = \frac{C(x)}{x} = \frac{0.2x^2 + 50x + 8000}{x} = 0.2x + 50 + \frac{8000}{x} \text{ (M1)}$$

Differentiating $A(x)$:

$$A'(x) = 0.2 - \frac{8000}{x^2} \text{ A1}$$

Set the derivative to zero to find the minimum:

$$0.2 - \frac{8000}{x^2} = 0 \implies 0.2x^2 = 8000 \implies x^2 = 40000 \implies x = 200 \text{ AG}$$

2. Minimum Average Cost:

$$A(200) = 0.2(200) + 50 + \frac{8000}{200} = 40 + 50 + 40 = \text{EUR } 130 \text{ A1}$$

3. Show Profit Function $\Pi(x)$:Revenue is Price $\times$ Quantity:

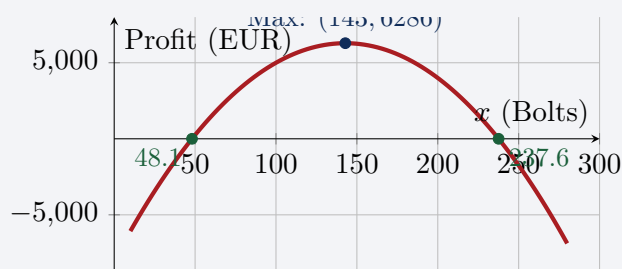
$$R(x) = x \cdot P(x) = x(250 - 0.5x) = 250x - 0.5x^2 \text{ (M1)}$$

Profit is Revenue $-$ Cost:

$$\Pi(x) = R(x) - C(x) = (250x - 0.5x^2) - (0.2x^2 + 50x + 8000) \text{ (A1)}$$

$$\Pi(x) = -0.5x^2 - 0.2x^2 + 250x - 50x - 8000 = -0.7x^2 + 200x - 8000 \text{ AG}$$

4. Profit Graph Sketch:



Marks: Convex-down quadratic shape [A1], vertex marked correctly [A1], roots labeled [A1].

5. Domain for Positive Profit:

$$\text{Set } \Pi(x) > 0 \implies -0.7x^2 + 200x - 8000 > 0.$$

Solving using the polynomial solver on GDC:

$$x_1 \approx 48.096, \quad x_2 \approx 237.618 \text{ (M1)(A1)}$$

Since x must be an integer, we restrict the bounds inside the interval:

$$49 \leq x \leq 237 \text{ A1}$$

Note: Award at most M1A1A0 if candidate rounds mathematically (e.g. 48 to 238) which would cause negative profit.

6. Max Profit and Optimal Selling Price:

$$\text{Maximum occurs at the vertex: } x = \frac{-b}{2a} = \frac{-200}{2(-0.7)} \approx 142.857 \approx 143 \text{ bolts. (M1)}$$

$$\text{Maximum Profit } \Pi(142.857) = -0.7(142.857)^2 + 200(142.857) - 8000 \approx \text{EUR } 6285.71.$$

A1

$$\text{Optimal selling price: } P(142.857) = 250 - 0.5(142.857) \approx \text{EUR } 178.57 \text{ per bolt. A1}$$



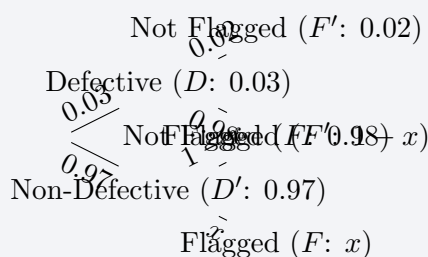
Examiner Tip & Common Trap

In optimization models, students frequently make the error of stating the optimal number of items (143 bolts) as their final price answer. Always read the question carefully to identify whether the target variable is quantity, total profit, or the price per unit.



Worked Solution & Mark Distribution - Question 4

1. Tree Diagram Construction:



Marks: Award A1 for primary branches (0.03, 0.97), A1 for defective sub-branches (0.98, 0.02), and A1 for non-defective sub-branches (x , $1 - x$).

2. Show that $x = 0.05$:

Using the total probability of being flagged:

$$P(F) = P(D \cap F) + P(D' \cap F) = 0.0779 \text{ (M1)}$$

$$(0.03 \times 0.98) + (0.97 \times x) = 0.0779 \text{ (A1)}$$

$$0.0294 + 0.97x = 0.0779 \implies 0.97x = 0.0485 \implies x = 0.05 \text{ AG}$$

3. Conditional Probability $P(D' | F)$:

We want the probability that a unit is non-defective, given that it was flagged:

$$P(D' | F) = \frac{P(D' \cap F)}{P(F)} = \frac{0.97 \times 0.05}{0.0779} \text{ (M1)(A1)}$$

$$P(D' | F) = \frac{0.0485}{0.0779} = 0.622593... \implies P(D' | F) \approx 0.623 \text{ A1}$$

4. Defect Rate in Shipped Units $P(D | F')$:

Shipped units are those not flagged (F'). Total probability of not being flagged:

$$P(F') = 1 - P(F) = 1 - 0.0779 = 0.9221 \text{ (A1)}$$

$$P(D | F') = \frac{P(D \cap F')}{P(F')} = \frac{0.03 \times 0.02}{0.9221} \text{ (M1)}$$

$$P(D | F') = \frac{0.0006}{0.9221} = 0.000650688... \implies P(D | F') \approx 0.000651 \text{ (or } 0.0651\%) \text{ A1}$$

5. Binomial Sample on Shipped Units:

Let Y be the number of defective units in a sample of 15 where $Y \sim B(15, p_{\text{shipped}} = 0.000650688)$:

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - 0.000650688)^{15} \text{ (M1)(A1)}$$

$$P(Y \geq 1) = 1 - (0.999349)^{15} = 1 - 0.99028... = 0.009715...$$

$$P(Y \geq 1) \approx 0.00972 \text{ (or } 0.972\%) \text{ A1A1}$$



Examiner Tip & Common Trap

When moving from standard conditional probability to sequential sampling (part 5), verify whether the sample is drawn from the *general* population ($p = 0.03$) or from the filtered *shipped* population ($p \approx 0.000651$). Misidentifying the binomial probability parameter will invalidate the entire final section.



Worked Solution & Mark Distribution - Question 5

1. Elena's Week 15 Target (u_{15}):

Arithmetic sequence where $u_1 = 30\,000$ and $d = 2500$:

$$u_{15} = u_1 + 14d = 30\,000 + 14(2500) \text{ (M1)}$$

$$u_{15} = 30\,000 + 35\,000 = 65\,000 \text{ steps (A1)}$$

2. Elena's Cumulative Steps (S_{15}):

Using the arithmetic sum formula:

$$S_{15} = \frac{15}{2}[2u_1 + 14d] = 7.5 \times [2(30\,000) + 14(2500)] \text{ (M1)(A1)}$$

$$S_{15} = 7.5 \times [60\,000 + 35\,000] = 7.5 \times 95\,000 = 712\,500 \text{ steps (A1)}$$

3. Kenji's Week 15 Target (v_{15}):

Geometric sequence where $v_1 = 20\,000$ and $r = 1.12$:

$$v_{15} = v_1 \times r^{14} = 20\,000 \times (1.12)^{14} \text{ (M1)}$$

$$v_{15} = 20\,000 \times 4.88711... = 97\,742.18... \implies v_{15} \approx 97\,742 \text{ steps (A1)}$$

4. Kenji's Cumulative Steps (S'_{15}):

Using the geometric sum formula:

$$S'_{15} = \frac{v_1(r^{15} - 1)}{r - 1} = \frac{20\,000(1.12^{15} - 1)}{1.12 - 1} \text{ (M1)(A1)}$$

$$S'_{15} = \frac{20\,000(5.473566... - 1)}{0.12} = 745\,594.3... \implies S'_{15} \approx 745\,594 \text{ steps (A1)}$$

5. Individual Target Crossover Week (n):

We solve the inequality: $20\,000(1.12)^{n-1} > 30\,000 + 2500(n-1)$. (M1)

Using GDC graph intersection or table scanning:

$$\text{At } x = 9: \quad \text{Elena} = 50\,000, \quad \text{Kenji} \approx 49\,519 \text{ (A1)}$$

$$\text{At } x = 10: \quad \text{Elena} = 52\,500, \quad \text{Kenji} \approx 55\,462 \text{ (A1)}$$

Therefore, Kenji's weekly target first strictly exceeds Elena's in **Week 10**. (A1)

6. Cumulative Target Crossover Week (k):

We solve the sum inequality: $\frac{20\,000(1.12^k - 1)}{0.12} > \frac{k}{2}[2(30\,000) + 2500(k-1)]$. (M1)

Using GDC to generate cumulative lists/sums:

$$\text{At } x = 13: \quad \text{Elena Sum} = 585\,000, \quad \text{Kenji Sum} \approx 560\,582 \text{ (A1)}$$

$$\text{At } x = 14: \quad \text{Elena Sum} = 647\,500, \quad \text{Kenji Sum} \approx 647\,852 \text{ (A1)}$$

Therefore, Kenji's cumulative steps first strictly exceed Elena's in **Week 14**. (A1)

Examiner Tip & Common Trap

When analyzing series crossover thresholds, there is a fundamental difference between individual terms exceeding each other (part 5) versus accumulated sums exceeding each other (part 6). Always write down the specific inequality or values of the sequence elements immediately surrounding the boundary to secure method marks.

