

IB Maths AI HL Unit 5 Calculus Practice Exam

IB Mathematics: Applications and Interpretation HL · 13 questions · 58 marks

Name _____

Date _____

Score / 58

- Answer every question. Show your working: most marks are for method.
- Calculator allowed.
- Topics: Systems and Phase Planes; Integration Methods; Geometric Applications; Kinematics Problems; Solving First Order ODEs; Phase Trajectories and Fields; Euler's Method Approximation.
- The mark scheme is a separate free PDF: ibmathrevision.com/downloads/practice-exams/hlai/ib-maths-ai-hl-unit-5-calculus-practice-exam-mark-scheme.pdf

1. Write the differential equation $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$ as a system of two coupled first-order differential equations. [3]

2. Find $\int \left(3\sqrt{x} - \frac{2}{x} + e^x \right) dx$, for $x > 0$. [3]

3. The region under the curve $y = e^{0.5x}$, between $x = 0$ and $x = 2$, is rotated through 2π about the x -axis. [3]
Find the volume of the solid formed.

4. The acceleration of a particle moving in a straight line is $a(t) = 6t - 4 \text{ m s}^{-2}$. When $t = 0$ the velocity is 1 m s^{-1} . [5]
- Find $v(t)$. [2]
 - Find the times when the particle is instantaneously at rest. [1]
 - Find the displacement of the particle between $t = 0$ and $t = 2$. [2]



5. Solve the differential equation $\frac{dy}{dx} = xy^2$, given that $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$, and hence find y when $x = 1$. [6]

6. Consider the differential equation $\frac{dy}{dx} = x + y$. [4]
- a. Find the equation of the isocline on which the segments of the slope field have gradient 0, and the isocline on which they have gradient 1. [2]
- b. Show that the line $y = -x - 1$ is a solution curve of the differential equation. [2]

7. Consider $\frac{d^2x}{dt^2} = t - \frac{dx}{dt}$, with $x = 0$ and $\frac{dx}{dt} = 1$ when $t = 0$. [3]
- a. Use Euler's method with step length 0.2 to find an approximate value of x when $t = 0.4$. [3]

8. A system is given by $\frac{dx}{dt} = y$, $\frac{dy}{dt} = -9x - 2y$. [4]
- a. Write down the second-order differential equation for x . [1]
- b. Find the eigenvalues of the system, and classify the equilibrium at the origin. [3]

9. This question uses integration by inspection. [4]
- a. Find $\int \frac{x}{x^2 + 1} dx$. [2]
- b. Hence find the exact value of $\int_0^{\sqrt{3}} \frac{x}{x^2 + 1} dx$. [2]



10. The region R is bounded by the curve $y = x^2$, the y -axis and the line $y = 4$, for $x \geq 0$. [5]

- Find the area of R by integrating with respect to y . [2]
- The region R is rotated through 2π about the y -axis. Find the volume of the solid formed. [3]

11. A cyclist moves along a straight road with velocity $v(t) = 10 - 3\sqrt{t} \text{ m s}^{-1}$, for $0 \leq t \leq 16$. She starts at the point O . [8]

- Find the time at which she changes direction. [2]
- Find her displacement from O at time t . [2]
- Find her greatest distance from O , and the total distance she travels in the 16 seconds. [4]

12. Consider the differential equation $\frac{dy}{dx} = y - x^2$. [5]

- Find the equation of the curve on which the segments of the slope field are horizontal. [1]
- Describe the region of the xy -plane in which solution curves are increasing. [2]
- A solution curve passes through $(0, 1)$. Find the gradient of the curve at this point, and state whether the curve is increasing or decreasing there. [2]



- 13.** The angle θ radians of a pendulum from the vertical satisfies $\frac{d^2\theta}{dt^2} = -9.8 \sin \theta$. At $t = 0$, $\theta = 0.5$ and $\frac{d\theta}{dt} = 0$. [5]
- a. Write the equation as a system of first-order equations. [1]
- b. Use Euler's method with step length 0.1 to estimate θ and $\frac{d\theta}{dt}$ when $t = 0.3$. [4]

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