

IB Maths AI HL Unit 5 Calculus Practice Exam: Mark Scheme

IB Mathematics: Applications and Interpretation HL · 13 questions · 58 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; R marks are for clear reasoning.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: ibmathrevision.com/HLAIUnit5

Question 1

3 marks

$$\text{let } y = \frac{dx}{dt} \text{ M1}$$

$$\frac{dx}{dt} = y \text{ A1}$$

$$\frac{dy}{dt} = -2x - 3y \text{ A1}$$

Question 2

3 marks

$$3 \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = 2x^{\frac{3}{2}} \text{ A1}$$

$$-2 \ln x \text{ A1}$$

$$+ e^x + C \text{ A1}$$

Question 3

3 marks

$$V = \pi \int_0^2 (e^{0.5x})^2 dx = \pi \int_0^2 e^x dx \text{ M1A1}$$

$$= \pi(e^2 - 1) = 20.1 \text{ A1}$$

Question 4

5 marks

a. $v = 3t^2 - 4t + C \text{ M1}$

$$v = 3t^2 - 4t + 1 \text{ A1}$$

b. $t = \frac{1}{3}$ and $t = 1 \text{ A1}$

c. $[t^3 - 2t^2 + t]_0^2 \text{ M1}$
 $= 2 \text{ m A1}$

Question 5

6 marks

$$\int y^{-2} dy = \int x dx \text{ M1}$$

$$-\frac{1}{y} = \frac{x^2}{2} + c \text{ A1A1}$$

$$c = -1 \text{ A1}$$

$$y = \frac{2}{2 - x^2} \text{ A1}$$

$$y(1) = 2 \text{ A1}$$



Question 6

4 marks

a. $y = -x$ **A1**

$y = 1 - x$ **A1**

b. on $y = -x - 1$: $\frac{dy}{dx} = x + (-x - 1) = -1$ **M1**

which equals the gradient of the line, so the line follows the slope field **R1****Question 7**

3 marks

a. $\frac{dx}{dt} = y$, $\frac{dy}{dt} = t - y$ **M1**

$t = 0.2$: $x = 0.200$, $y = 0.800$ **A1**

$t = 0.4$: $x \approx 0.360$ **A1**

Question 8

4 marks

a. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 9x = 0$ **A1**

b. $\lambda^2 + 2\lambda + 9 = 0$ **M1**

$\lambda = -1 \pm 2\sqrt{2}i$ **A1**

stable spiral **A1****Question 9**

4 marks

a. the numerator is half the derivative of $x^2 + 1$ **M1**

$\frac{1}{2}\ln(x^2 + 1) + C$ **A1**

b. $\frac{1}{2}\ln 4 - \frac{1}{2}\ln 1$ **M1**

$= \ln 2$ **A1**

Question 10

5 marks

a. $\int_0^4 \sqrt{y} dy$ **M1**

$= \frac{16}{3}$ **A1**

b. $V = \pi \int_0^4 x^2 dy = \pi \int_0^4 y dy$ **M1A1**

$= 8\pi = 25.1$ **A1**

Question 11

8 marks

a. $3\sqrt{t} = 10$ **M1**

$t = \frac{100}{9} = 11.1$ s **A1**

b. $s = 10t - 2t^{\frac{3}{2}}$ **A1A1**

c. $s\left(\frac{100}{9}\right) = \frac{1000}{27} = 37.0$ m **A1**

$s(16) = 32$ **A1**

total $2 \times \frac{1000}{27} - 32 = 42.1$ m **M1A1**



Question 12**5 marks**

a. $y = x^2$ **A1**

b. $y - x^2 > 0$ **M1**

the region above the parabola $y = x^2$ **A1**

c. $1 - 0 = 1$ **A1**

increasing **A1**

Question 13**5 marks**

a. $\frac{d\theta}{dt} = \omega, \frac{d\omega}{dt} = -9.8 \sin \theta$ **A1**

b. $\theta_{n+1} = \theta_n + 0.1\omega_n, \omega_{n+1} = \omega_n - 0.98 \sin \theta_n$ **M1**

$t = 0.1: \theta = 0.5000, \omega = -0.4698$ **A1**

$t = 0.2: \theta = 0.4530, \omega = -0.9397$ **A1**

$t = 0.3: \theta \approx 0.359, \frac{d\theta}{dt} \approx -1.37$ **A1**

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