

IB Maths AI HL Unit 3 Geometry and Trigonometry Practice Exam

IB Mathematics: Applications and Interpretation HL · 14 questions · 60 marks

Name _____

Date _____

Score / 60

- Answer every question. Show your working: most marks are for method.
- Calculator allowed.
- Topics: Fundamental Ratios and Pythagoras; Arcs and Sectors; Advanced Trigonometry; Geometric Matrix Transformations; Vector Geometry and Algebra; Constant Velocity Kinematics; Graph Properties and Walks; Adjacency Matrices; Sine Rule applications; Area formula applications; Graph Algorithms.
- The mark scheme is a separate free PDF: ibmathrevision.com/downloads/practice-exams/hlai/ib-maths-ai-hl-unit-3-geometry-trigonometry-practice-exam-mark-scheme.pdf

1. Write down the exact values of $\sin \frac{\pi}{6}$, $\cos \frac{\pi}{3}$ and $\tan \frac{\pi}{4}$. [2]
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2. A pendulum of length 80 cm swings through an angle of 0.3 radians either side of the vertical. [3]
 - a. Find the length of the arc travelled by the end of the pendulum in one swing from one side to the other. [2]
 - b. Express the total angle in degrees. [1]
|
3. Use technology to solve the equation $3 \sin(2x) = x$ for $0 \leq x \leq \pi$. [3]
|
4. Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to show that $\frac{1 - \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta$, for $\sin \theta \cos \theta \neq 0$. [2]
|



5. Triangle T has vertices $(0, 0)$, $(4, 0)$ and $(1, 3)$. It is transformed by the matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$. [5]
- Find the coordinates of the vertices of the image of T . [2]
 - Find the area of T and hence the area of its image. [3]
6. A boat is steered due north at 6 km h^{-1} relative to the water, while a current flows due east at 2.5 km h^{-1} . Let $\mathbf{i}$ be a unit vector east and $\mathbf{j}$ a unit vector north. [4]
- Write down the resultant velocity of the boat as a column vector. [1]
 - Find the resultant speed and the bearing on which the boat travels. [3]
7. An aircraft flies in a straight line from $P(10, 20, 3)$ towards $Q(70, 100, 3)$, where distances are in km, at a constant speed of 250 km h^{-1} . [5]
- Find a unit vector in the direction of travel. [2]
 - Write down the velocity vector, and an equation for the position of the aircraft t hours after it passes P . [2]
 - Find the time taken to fly from P to Q , in minutes. [1]
8. A connected graph has six vertices, with degrees 2, 4, 3, 3, 2, 4. [3]
- Find the number of edges. [1]
 - State, with a reason, whether the graph has an Eulerian circuit, an Eulerian trail, or neither. [2]



9. A graph with vertices P, Q, R, S , in that order, has adjacency matrix $M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$. [4]

- Find M^3 . [1]
- Find the number of walks of length 3 from P to S . [1]
- Find the number of walks of length at most 3 from Q back to Q . [2]

10. A bicycle wheel of radius 0.35 m turns through 5 complete revolutions every 2 seconds. The wheel rolls without slipping. [4]

- Find the angle, in radians, turned through by the wheel in one second. [1]
- Find the speed of the bicycle in km h^{-1} . [3]

11. In triangle PQR , $PQ = 9$ cm, $QR = 6$ cm and $\hat{QPR} = 35^\circ$. [5]

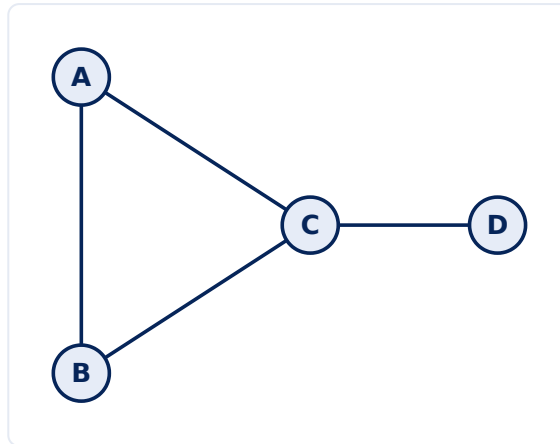
- Find the two possible values of $\hat{PRQ}$. [3]
- Find the smallest possible length of PR . [2]

12. A triangle ABC has $AB = 12$ cm, $AC = 8$ cm and area 30 cm^2 . [5]

- Find the two possible values of $\hat{BAC}$, in degrees. [3]
- Find BC when $\hat{BAC}$ is obtuse. [2]



- 13.** A graph has vertices A, B, C, D and edges AB, AC, BC, CD . A robot moves along the edges: at each step it leaves its current vertex along one of the edges at that vertex, each being equally likely. [7]
- a. Write down the transition matrix T , with columns giving the current vertex and rows the next vertex, in the order A, B, C, D . [2]
- b. The robot starts at A . Find the probability that it is at D after two steps. [2]
- c. Show that $\mathbf{s} = \frac{1}{8} \begin{pmatrix} 2 \\ 2 \\ 3 \\ 1 \end{pmatrix}$ is a steady state of T , and interpret it. [3]



- 14.** A delivery driver must visit five towns P, Q, R, S, T , starting and finishing at P . The table gives the travel times, in minutes, between each pair of towns. [8]

	P	Q	R	S	T
P	–	12	9	15	11
Q	12	–	8	13	14
R	9	8	–	10	16
S	15	13	10	–	7
T	11	14	16	7	–

- a. Use the nearest neighbour algorithm, starting at P , to find an upper bound for the minimum total travel time. State the route. [3]
- b. By deleting vertex P , use the deleted vertex algorithm to find a lower bound for the minimum total travel time. [4]
- c. Write down an inequality for the minimum total travel time L . [1]



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