

IB Maths AI HL Unit 3 Geometry and Trigonometry Practice Exam: Mark Scheme

IB Mathematics: Applications and Interpretation HL · 14 questions · 60 marks

- M marks are for a correct method; A marks for an accurate answer that follows a correct method; R marks are for clear reasoning; AG means the answer is given in the question, so the working must be complete.
- Accept any correct method that reaches a correct answer, unless the question says which method to use.
- Practise these topics online with instant feedback: ibmathrevision.com/HLAIUnit3

Question 1

2 marks

$\frac{1}{2}$, $\frac{1}{2}$, 1 (A1 for any two correct) **A2**

Question 2

3 marks

a. total angle 0.6 radians **(A1)**

$$80 \times 0.6 = 48 \text{ cm } \mathbf{A1}$$

b. $0.6 \times \frac{180}{\pi} = 34.4^\circ$ **A1**

Question 3

3 marks

graphs of $y = 3 \sin(2x)$ and $y = x$, or of $y = 3 \sin(2x) - x$, on the given domain **(M1)**

$$x = 0 \text{ } \mathbf{A1}$$

$$x = 1.34 \text{ } \mathbf{A1}$$

Question 4

2 marks

$$1 - \cos^2 \theta = \sin^2 \theta \text{ } \mathbf{M1}$$

$$\frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} \text{ } \mathbf{A1}$$

$$= \tan \theta \text{ } \mathbf{AG}$$

Question 5

5 marks

a. (0, 0), (8, 4) **A1**

(5, 10) **A1**

b. area of $T = \frac{1}{2}(4)(3) = 6$ **A1**

$$\det M = 5 \text{ } \mathbf{M1}$$

$$\text{image area} = 5 \times 6 = 30 \text{ } \mathbf{A1}$$

Question 6

4 marks

a. $\begin{pmatrix} 2.5 \\ 6 \end{pmatrix}$ **A1**

b. $\sqrt{2.5^2 + 6^2} = 6.5 \text{ km h}^{-1}$ **A1**

$$\arctan \frac{2.5}{6} \text{ } \mathbf{M1}$$

$$\text{bearing } 022.6^\circ \text{ } \mathbf{A1}$$



Question 7**5 marks**

a. $\overrightarrow{PQ} = \begin{pmatrix} 60 \\ 80 \\ 0 \end{pmatrix}$, length 100 **A1**

$\begin{pmatrix} 0.6 \\ 0.8 \\ 0 \end{pmatrix}$ **A1**

b. $\mathbf{v} = \begin{pmatrix} 150 \\ 200 \\ 0 \end{pmatrix}$ **A1**

$\mathbf{r} = \begin{pmatrix} 10 \\ 20 \\ 3 \end{pmatrix} + t \begin{pmatrix} 150 \\ 200 \\ 0 \end{pmatrix}$ **A1**

c. $\frac{100}{250} \text{ h} = 24 \text{ minutes}$ **A1**

Question 8**3 marks**

a. $\frac{18}{2} = 9$ **A1**

b. exactly two vertices have odd degree **R1**

an Eulerian trail (not a circuit), starting and ending at the two odd vertices **A1**

Question 9**4 marks**

a. $\begin{pmatrix} 2 & 5 & 5 & 2 \\ 5 & 4 & 5 & 5 \\ 5 & 5 & 4 & 5 \\ 2 & 5 & 5 & 2 \end{pmatrix}$ **A1**

b. 2 **A1**

c. entry (2,2) of $M + M^2 + M^3$ **M1**

7 **A1**

Question 10**4 marks**

a. $\frac{5 \times 2\pi}{2} = 5\pi$ **A1**

b. arc length per second = $0.35 \times 5\pi$ **(M1)**

= 5.50 m s^{-1} **A1**

= 19.8 km h^{-1} **A1**

Question 11**5 marks**

a. $\frac{\sin R}{9} = \frac{\sin 35^\circ}{6}$ **M1**

$R = 59.4^\circ$ **A1**

or $R = 180^\circ - 59.4^\circ = 121^\circ$ **A1**

b. obtuse case: $\hat{PQR} = 180^\circ - 35^\circ - 121^\circ = 24.4^\circ$ **(M1)**

$PR = \frac{6 \sin 24.4^\circ}{\sin 35^\circ} = 4.31 \text{ cm}$ **A1**



Question 12

5 marks

- a. $\frac{1}{2}(12)(8) \sin A = 30$ **M1**
 $\sin A = 0.625 \implies A = 38.7^\circ$ **A1**
 or $A = 141^\circ$ **A1**
- b. $BC^2 = 12^2 + 8^2 - 2(12)(8) \cos A$ **(M1)**
 $BC = 18.9$ cm **A1**

Question 13

7 marks

- a. $T = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ **A1A1**
- b. $T^2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, or $A \rightarrow C \rightarrow D: \frac{1}{2} \times \frac{1}{3}$ **M1**
 $\frac{1}{6}$ **A1**
- c. $Ts = s$, checked row by row **M1**
 entries are non-negative and sum to 1 **A1**
 in the long run the robot is at C 37.5% of the time (proportional to degree) **R1**

Question 14

8 marks

- a. $P \rightarrow R (9) \rightarrow Q (8) \rightarrow S (13) \rightarrow T (7) \rightarrow P (11)$ **M1A1**
 upper bound 48 minutes **A1**
- b. minimum spanning tree of Q, R, S, T : $ST (7), QR (8), RS (10)$ **M1**
 weight 25 **A1**
 two shortest edges from P : $PR (9) + PT (11)$ **M1**
 lower bound 45 minutes **A1**
- c. $45 \leq L \leq 48$ **A1**

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