

IB Maths AI HL Unit 2 Functions Practice Exam

IB Mathematics: Applications and Interpretation HL · 12 questions · 60 marks

Name _____

Date _____

Score / 60

- Answer every question. Show your working: most marks are for method.
- Calculator allowed. Includes core questions from the SL AI unit, which HL students also sit.
- Topics: Function Transformations; Constructing Models; Graph properties and sketching; Analyzing Given Models.
- The mark scheme is a separate free PDF: ibmathrevision.com/downloads/practice-exams/hlai/ib-maths-ai-hl-unit-2-functions-practice-exam-mark-scheme.pdf

1. The graph of $y = f(x)$ passes through the point $(2, 5)$. Write down the image of this point under each transformation. [4]

a. $y = f(x) + 3$ [1]

b. $y = f(x - 4)$ [1]

c. $y = 2f(x)$ [1]

d. $y = f(-x)$ [1]

2. Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of each of the following. [3]

a. $y = f\left(\frac{x}{3}\right)$ [2]

b. $y = -f(x)$ [1]

3. A student believes that the period P seconds of a pendulum is proportional to $\sqrt{L}$, where L m is its length. [3]

L	0.25	0.5	1.0	1.5	2.0
P	1.0	1.42	2.01	2.46	2.84

a. Calculate $\frac{P}{\sqrt{L}}$ for each pair of values. [1]

b. Hence comment on the student's belief and write down a model for P . [2]



4. The graph of $y = x^2$ is translated by $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and then stretched vertically by scale factor 4. [4]

- Find the equation of the resulting graph. [2]
- Write down the coordinates of its vertex. [1]
- State whether performing the stretch first would give the same graph, justifying your answer. [1]

5. The temperature, in $^{\circ}\text{C}$, t hours after midnight on a spring day is modelled by $T(t) = 12 + 6 \sin(15t)^{\circ}$. A model for a day in a warmer month is $W(t) = T(t - 2) + 4$. [5]

- Describe the two transformations that map the graph of T onto the graph of W . [2]
- Find the maximum value of W and the first time $t > 0$ at which it occurs. [3]

6. Let $g(x) = 2(1.5)^x$ and $h(x) = 8 - x$. [5]

- On the same axes, sketch the graphs of g and h for $-2 \leq x \leq 6$. [2]
- Find the coordinates of the point of intersection. [2]
- Write down the equation of the horizontal asymptote of the graph of g . [1]

7. The number of users N (thousands) of an app t months after launch is shown. [6]

t	0	1	2	3	4	5
N	50	68	91	124	166	225

- Calculate the ratios $\frac{N(t+1)}{N(t)}$ and use them to suggest whether a linear or an exponential model is more appropriate. [2]
- Use your GDC to find an exponential model $N = ab^t$. [2]
- Use your model to estimate the number of users after 8 months, and comment on its reliability. [2]



8. A model for the number of visitors V (hundreds) to a museum in month m is [5]
 $V(m) = -0.5m^2 + 6m + 10$, $1 \leq m \leq 12$. The actual numbers in months 2, 6 and 10 were 19, 28 and 22 hundred.
- Calculate the values predicted by the model for months 2, 6 and 10. [2]
 - Calculate the percentage error of the model in month 10. [2]
 - Comment on how well the model fits the data. [1]
9. Let $f(x) = x^3 - 3x$. The graph of $y = g(x)$ is obtained from the graph of $y = f(x)$ by a reflection in the y [5]
 $-$ -axis followed by a translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.
- Find $g(x)$. [2]
 - The point $(1, -2)$ is a local minimum of f . Find the image of this point on the graph of g , and state its nature. [3]
10. The height, h metres, of a ball t seconds after it is thrown is modelled by $h(t) = -4.9t^2 + 14t + 1.5$, for [8]
 $0 \leq t \leq T$, where $h(T) = 0$.
- Find the value of T . [1]
 - Find the maximum height of the ball and the time at which it occurs. [2]
 - Sketch the graph of h against t , labelling the intercepts and the maximum point. [2]
 - Find the length of time for which the ball is at least 8 m above the ground. [3]



- 11.** The depth of water, D metres, at the entrance to a harbour t hours after midnight was recorded. [6]

t	0	2	4	6	8	10	12
D (m)	7.4	6.0	3.2	1.8	3.2	6.0	7.4

The depth is modelled by $D(t) = a \cos(bt) + d$, where t is in hours and bt is in degrees.

- Write down the period of the model and hence show that $b = 30$. [2]
- Use the maximum and minimum depths to find the values of a and d . [2]
- A boat needs a depth of at least 3.5 m to enter the harbour. Find the times, for $0 \leq t \leq 12$, during which the boat cannot enter. [2]

- 12.** The number of subscribers, in thousands, to a streaming service t months after launch is modelled by [6]

$$N(t) = \frac{80}{1 + 15e^{-0.4t}}. \text{ A rival service is modelled by } R(t) = 1.5N(t - 3).$$

- Describe the transformations that map the graph of $y = N(t)$ onto the graph of $y = R(t)$. [2]
- Write down the equation of the horizontal asymptote of the graph of R , and interpret it in context. [2]
- Find the value of t at which the two services have the same number of subscribers. [2]

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